

9.

Figure 3

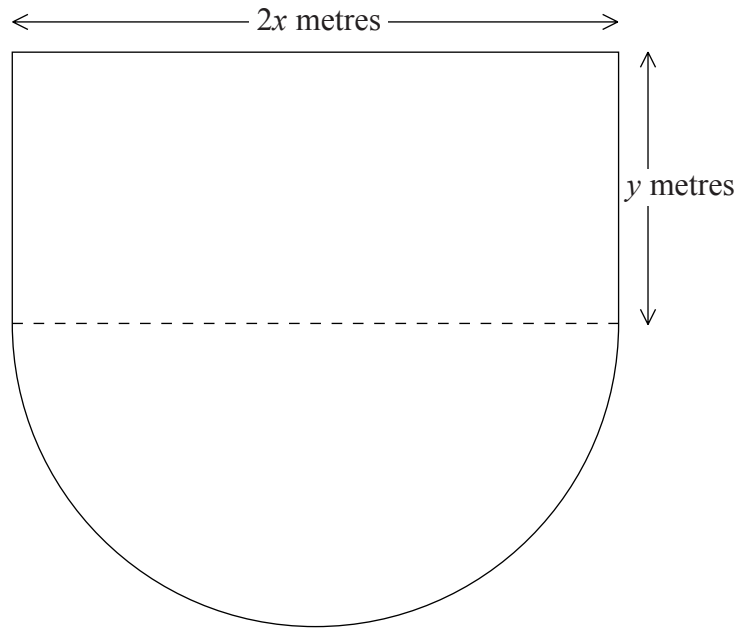


Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(a) Show that the area, A m², of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2. \quad (4)$$

(b) Use calculus to find the value of x at which A has a stationary value. (4)

(c) Prove that the value of x you found in part (b) gives the maximum value of A . (2)

(d) Calculate, to the nearest m², the maximum area of the stage. (2)

8.

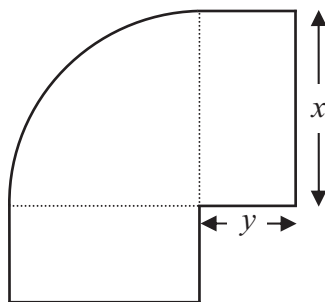


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2 ,

(a) show that

$$y = \frac{16 - \pi x^2}{8x} \tag{3}$$

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x \tag{3}$$

(c) Use calculus to find the minimum value of P . (5)

(d) Find the width of each rectangle when the perimeter is a minimum. Give your answer to the nearest centimetre. (2)



9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75\pi\text{ cm}^3$.

The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is $r\text{ cm}$,

- (a) show that the cost of the polishing, $\pounds C$, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r} \qquad (4)$$

- (b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound. (5)

- (c) Justify that the answer that you have obtained in part (b) is a minimum. (1)



9.

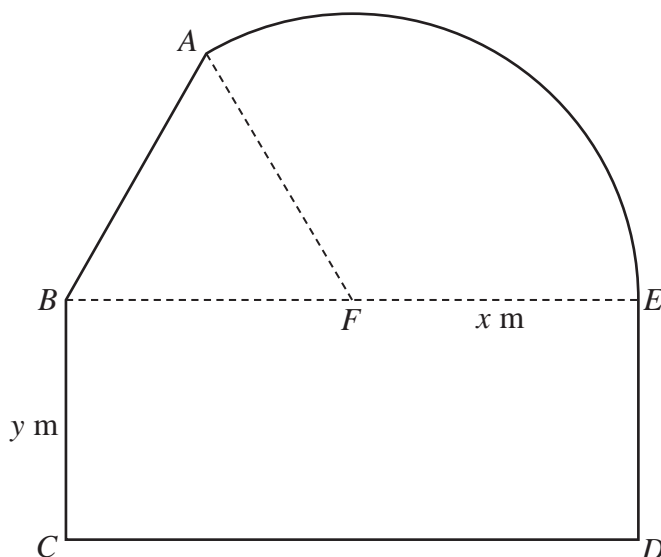


Diagram not drawn to scale

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure $ABCDEA$, as shown in Figure 4, consists of a rectangle $BCDE$ joined to an equilateral triangle BFA and a sector FEA of a circle with radius x metres and centre F .

The points B, F and E lie on a straight line with $FE = x$ metres and $10 \leq x \leq 25$

- (a) Find, in m^2 , the exact area of the sector FEA , giving your answer in terms of x , in its simplest form. (2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2 ,

- (b) show that

$$y = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})$$
(3)

- (c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3})$$
(3)

- (d) Use calculus to find the minimum value of P , giving your answer to the nearest metre. (5)

- (e) Justify, by further differentiation, that the value of P you have found is a minimum. (2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

