Basic Differentiation Exercise 12E Page 267

4) i $\quad \frac{d}{dx} \frac{2x^2+3x}{\sqrt{x}} = \frac{d}{dx}\left(2x^{3/2} + 3x^{\frac{1}{2}}\right)$

$= 3x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$

4 f) $\quad \sqrt[3]{x} + \frac{1}{2x} = x^{\frac{1}{3}} + \frac{1}{2}x^{-1}$

$\frac{d}{dx}\left(x^{\frac{1}{3}} + \frac{1}{2}x^{-1}\right) = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$

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4 g) $\quad \frac{2x+3}{x} = 2 + \frac{3}{x} = 2 + 3x^{-1}$

$\frac{d}{dx}\left(2 + 3x^{-1}\right) = -3x^{-2}$

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4 h) $\quad \frac{3x^2-6}{x} = 3x - \frac{6}{x} = 3x - 6x^{-1}$

$\frac{d}{dx}\left(3x - 6x^{-1}\right) = 3 + 6x^{-2}$

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4 j) $\quad x\left(x^2 - x + 2\right) = x^3 - x^2 + 2x$

$$\frac{d}{dx}\left(x^3 - x^2 + 2x\right) = 3x^2 - 2x + 2$$

4k) $3x^2(x^2 + 2x) = 3x^4 + 6x^3$

$$\frac{d}{dx}\left(3x^4 + 6x^3\right) = 12x^3 + 18x^2$$

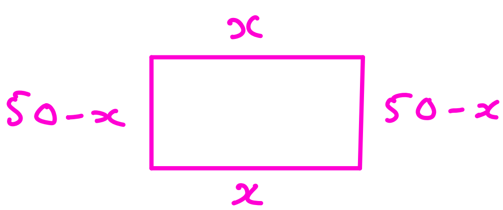
4e) $(3x+2)\left(4x + \frac{1}{x}\right) = 12x^2 + 8x + 3 + \frac{2}{x}$

$$\frac{d}{dx}\left(12x^2 + 8x + 3 + 2x^{-1}\right) = 24x + 8 - 2x^{-2}$$

Introduction to Modelling Using Differentiation

Ex1 100 m of fence for rectangular paddock

What should dimensions be to maximise area



Area $= x(50 - x)$

$A = 50x - x^2$

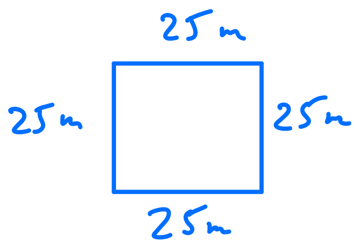
$\frac{dA}{dx} = 50 - 2x$

At a max or min $\frac{dA}{dx} = 0 \Rightarrow 50 - 2x = 0$

$$50 = 2x$$

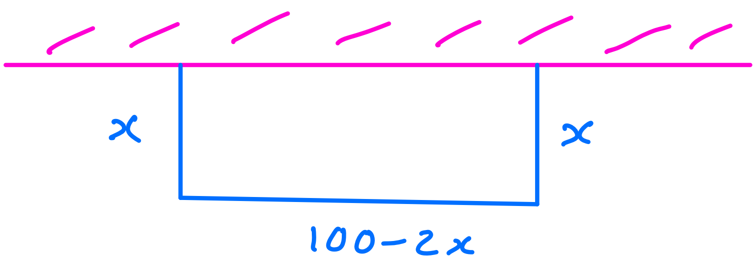
$$\underline{x = 25}$$

Check this is a max $\qquad \dfrac{d^2A}{dx^2} = -2 \quad \therefore$ a max



Area $25 \times 25 = 625\ m^2$

Ex 2 $\quad$ 100 m fence – build a rectangular paddock against wall and maximise area



Area $\qquad A = x(100 - 2x)$

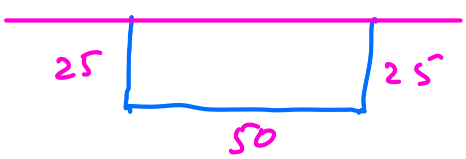
$$A = 100x - 2x^2$$

$$\frac{dA}{dx} = 100 - 4x$$

At t.p. $\qquad \dfrac{dA}{dx} = 0 \Rightarrow 100 - 4x = 0$

$$100 = 4x$$

$$\underline{x = 25\ m}$$



Area $= 50 \times 25 = 1250\ m^2$

Check a max $\frac{d^2A}{dx^2} = -4 < 0 \therefore$ a max