Question Number	Scheme		Marks
3. (a)	From question, $\frac{dA}{dt} = 0.032$	$\frac{dA}{dt} = 0.032$ seen or implied from working.	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} =\right\} 2\pi x$	$2\pi x$ by itself seen or implied from working	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}; \left\{= \frac{0.016}{\pi x}\right\}$	$0.032 \div$ Candidate's $\frac{dA}{dx}$;	M1;
	When $x = 2\,\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$		
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	$\frac{dV}{dx} = 15\pi x^2$ or ft from candidate's V **in one variable**	B1$\surd$
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2.\left(\frac{0.016}{\pi x}\right); \{= 0.24x\}$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$;	M1$\surd$
	When $x = 2\,\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ (cm^3 s^{-1})	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]
			8 marks

Question Number	Scheme		Marks
5. (a)	Similar triangles $\Rightarrow \frac{r}{h}=\frac{16}{24} \Rightarrow r=\frac{2h}{3}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
			[2]
(b)	From the question, $\frac{dV}{dt}=8$	$\frac{dV}{dt}=8$	B1
	$\frac{dV}{dh}=\frac{12\pi h^2}{27}=\frac{4\pi h^2}{9}$	$\frac{dV}{dh}=\frac{12\pi h^2}{27}$ or $\frac{4\pi h^2}{9}$	B1
	$\frac{dh}{dt}=\frac{dV}{dt}\div\frac{dV}{dh} = 8\times\frac{9}{4\pi h^2} = \frac{18}{\pi h^2}$	Candidate's $\frac{dV}{dt}\div\frac{dV}{dh}$; $8\div\left(\frac{12\pi h^2}{27}\right)$ or $8\times\frac{9}{4\pi h^2}$ or $\frac{18}{\pi h^2}$ oe	M1; A1
	When $h=12$, $\frac{dh}{dt} = \frac{18}{144\pi} = \frac{1}{8\pi}$	$\frac{18}{144\pi}$ or $\frac{1}{8\pi}$	A1 oe isw
			[5]
			7 marks

Note the answer must be a one term exact value.

Note, also you can ignore subsequent working after $\frac{18}{144\pi}$.

Question Number	Scheme	Marks
Q6	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \dfrac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$	B1
	When $A = 2$ $2 = \pi r^2 \Rightarrow r = \sqrt{\dfrac{2}{\pi}} \ \left(= 0.797\,884\ldots\right)$	M1
	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}r} \times \dfrac{\mathrm{d}r}{\mathrm{d}t}$ $1.5 = 2\pi r \dfrac{\mathrm{d}r}{\mathrm{d}t}$	M1
	$\dfrac{\mathrm{d}r}{\mathrm{d}t} = \dfrac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]

June 2011

Question Number	Scheme	Marks
3.	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$ or equivalent	M1 A1
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$ $\frac{\pi}{25}$	M1 A1 **(4)**
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$ or $\frac{\pi}{800} \div$ their (a)	M1
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$ awrt 0.031	A1 **(2)**
		[6]

June 2012

Question Number	Scheme	Marks
2.	(a) $V = x^3 \Rightarrow \dfrac{dV}{dx} = 3x^2$ * cso	B1 **(1)**
	(b) $\dfrac{dx}{dt} = \dfrac{dx}{dV} \times \dfrac{dV}{dt} = \dfrac{0.048}{3x^2}$	M1
	At $x = 8$	
	$\dfrac{dx}{dt} = \dfrac{0.048}{3\left(8^2\right)} = 0.00025 \quad \left(\text{cm s}^{-1}\right)$ 2.5×10^{-4}	A1 **(2)**
	(c) $S = 6x^2 \Rightarrow \dfrac{dS}{dx} = 12x$	B1
	$\dfrac{dS}{dt} = \dfrac{dS}{dx} \times \dfrac{dx}{dt} = 12x\left(\dfrac{0.048}{3x^2}\right)$	M1
	At $x = 8$	
	$\dfrac{dS}{dt} = 0.024 \quad \left(\text{cm}^2\,\text{s}^{-1}\right)$	A1 **(3)**
		[6]

Question Number	Scheme	Marks
4.	$\frac{dV}{dt} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$,	
	$\frac{dV}{dh} = 8\pi h + 16\pi$ $\quad \pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $\quad 8\pi h + 16\pi$	M1 A1
	$\left\{\frac{dV}{dh}\times\frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ (8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\quad \left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$	M1 oe
	$\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}\ \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$ $\quad$ or $\quad 80\pi \div \text{Candidate's } \frac{dV}{dh}$	
	When $h = 6$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi \left\{= \frac{80\pi}{64\pi}\right\}$ $\quad$ **dependent on the previous M1 see notes**	dM1
	$\frac{dh}{dt} = \underline{1.25}\ (\text{cm s}^{-1})$ $\quad 1.25$ or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	A1 oe
		[5] **5**
	Alternative Method for the first M1A1	
	Product rule: $\begin{cases} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{cases}$	
	$\frac{dV}{dh} = 4\pi(h+4) + 4\pi h$ $\quad \pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $\quad 4\pi(h+4) + 4\pi h$	M1 A1

Question 4 Notes

M1	An expression of the form $\pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$. **Can be simplified or un-simplified.**
A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent.
Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
Note	$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
M1	$\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$
Note	Also allow 2nd M1 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{dV}{dh}$
Note	Give 2nd M0 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{dV}{dh}$
dM1	**which is dependent on the previous M1 mark.** Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and 80π (or 80)
A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
Note	Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)