

More Binomial Questions

7 Any patient who fails to turn up for an outpatient appointment at a hospital is described as a 'no-show'. At a particular hospital, on average 15% of patients are no-shows. A random sample of 20 patients who have outpatient appointments is selected.

(i) Find the probability that

(A) there is exactly 1 no-show in the sample, [3]

(B) there are at least 2 no-shows in the sample. [2]

The hospital management introduces a policy of telephoning patients before appointments. It is hoped that this will reduce the proportion of no-shows. In order to check this, a random sample of n patients is selected. The number of no-shows in the sample is recorded and a hypothesis test is carried out at the 5% level.

(ii) Write down suitable null and alternative hypotheses for the test. Give a reason for your choice of alternative hypothesis. [4]

(iii) In the case that $n = 20$ and the number of no-shows in the sample is 1, carry out the test. [4]

(iv) In another case, where n is large, the number of no-shows in the sample is 6 and the critical value for the test is 8. Complete the test. [3]

(v) In the case that $n \leq 18$, explain why there is no point in carrying out the test at the 5% level. [2]

4 It is known that 8% of the population of a large city use a particular web browser. A researcher wishes to interview some people from the city who use this browser. He selects people at random, one at a time.

(i) Find the probability that the first person that he finds who uses this browser is

(A) the third person selected, [3]

(B) the second or third person selected. [2]

(ii) Find the probability that at least one of the first 20 people selected uses this browser. [3]

5 A manufacturer produces titanium bicycle frames. The bicycle frames are tested before use and on average 5% of them are found to be faulty. A cheaper manufacturing process is introduced and the manufacturer wishes to check whether the proportion of faulty bicycle frames has increased. A random sample of 18 bicycle frames is selected and it is found that 4 of them are faulty. Carry out a hypothesis test at the 5% significance level to investigate whether the proportion of faulty bicycle frames has increased. [8]

7 A coffee shop provides free internet access for its customers. It is known that the probability that a randomly selected customer is accessing the internet is 0.35, independently of all other customers.

(i) 10 customers are selected at random.

(A) Find the probability that exactly 5 of them are accessing the internet. [3]

(B) Find the probability that at least 5 of them are accessing the internet. [2]

(C) Find the expected number of these customers who are accessing the internet. [2]

Another coffee shop also provides free internet access. It is suspected that the probability that a randomly selected customer at this coffee shop is accessing the internet may be different from 0.35. A random sample of 20 customers at this coffee shop is selected. Of these, 10 are accessing the internet.

(ii) Carry out a hypothesis test at the 5% significance level to investigate whether the probability for this coffee shop is different from 0.35. Give a reason for your choice of alternative hypothesis. [9]

(iii) To get a more reliable result, a much larger random sample of 200 customers is selected over a period of time, and another hypothesis test is carried out. You are given that 90 of the 200 customers were accessing the internet. You are also given that, if X has the binomial distribution with parameters $n = 200$ and $p = 0.35$, then $P(X \geq 90) = 0.0022$. Using the same hypotheses and significance level which you used in part (ii), complete this test. [2]