

2 (a)	<p>Let X be the random variable the number of faulty bolts</p> $P(X \leq 2) - P(X \leq 1) = 0.0355 - 0.0076 \quad \text{or} \quad (0.3)^2(0.7)^{18} \frac{20!}{18!2!}$ $= 0.0279 \quad \quad \quad = 0.0278$	<p>M1 A1 (2) M1 A1 (2)</p>
(b)	$1 - P(X \leq 3) = 1 - 0.1071$ $= 0.8929$ <p>or $1 - (0.3)^3(0.7)^{17} \frac{20!}{17!3!} - (0.3)^2(0.7)^{18} \frac{20!}{18!2!} - (0.3)(0.7)^{19} \frac{20!}{19!1!} - (0.7)^{20}$</p>	<p>M1A1√A1 (3)</p>
(c)	$\frac{10!}{4!6!} (0.8929)^6 (0.1071)^4 = 0.0140.$	
Notes:		
2. (a)	<p>M1 Either attempting to use $P(X \leq 2) - P(X \leq 1)$ or attempt to use binomial and find $p(X = 2)$. Must have $(p)^2(1-p)^{18} \frac{20!}{18!2!}$, with a value of p</p> <p>A1 awrt 0.0278 or 0.0279.</p>	
(b)	<p>M1 Attempting to find $1 - P(X \leq 3)$</p> <p>A1 awrt 0.893</p>	
(c)	<p>M1 for $k (p)^k(1-p)^{n-k}$. They may use any value for p and k can be any number or ${}^nC_k p^k(1-p)^{n-k}$</p> <p>A1√ $\frac{10!}{4!6!} (\text{their part } b)^6 (1 - \text{their part } b)^4$ may write ${}^{10}C_6$ or ${}^{10}C_4$</p> <p>A1 awrt 0.014</p>	<p>B1 B1 (2)</p>

5	<p>$H_0 : p = 0.3; H_1 : p > 0.3$</p> <p>Let X represent the number of tomatoes greater than 4 cm : $X \sim B(40, 0.3)$</p> <p>$P(X \geq 18) = 1 - P(X \leq 17)$ $P(X \geq 18) = 1 - P(X \leq 17) = 0.0320$ $= 0.0320$ $P(X \geq 17) = 1 - P(X \leq 16) = 0.0633$ CR $X \geq 18$</p> <p>$0.0320 < 0.05$ $18 \geq 18$ or 18 in the critical region</p> <p>no evidence to Reject H_0 or it is significant</p> <p>New fertiliser has <u>increased</u> the probability of a <u>tomato</u> being greater than 4 cm Or Dhriti's claim is true</p>	<p>B1 B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1d cao (7)</p>
5	<p>B1 for correct H_0 . must use p or pi</p> <p>B1 for correct H_1 must use p and be one tail.</p> <p>B1 using B(40, 0.3). This may be implied by their calculation</p> <p>M1 attempt to find $1 - P(X \leq 17)$ or get a correct probability. For CR method must attempt to find $P(X \geq 18)$ or give the correct critical region</p> <p>A1 awrt 0.032 or correct CR.</p> <p>M1 correct statement based on their probability , H_1 and 0.05 or a correct contextualised statement that implies that.</p> <p>B1 this is not a follow through .conclusion in context. Must use the words increased, tomato and some reference to size or diameter. This is dependent on them getting the previous M1</p> <p>If they do a two tail test they may get B1 B0 B1 M1 A1 M1 B0 For the second M1 they must have accept H_0 or it is not significant or a correct contextualised statement that implies that.</p>	

<p>6a (i)</p> <p>ii)</p> <p>b)</p>	<p>Let X represent the number of sunflower plants more than 1.5m high</p> <p>$X \sim \text{Po}(10)$ $\mu=10$</p> <p>$P(8 \leq X \leq 13) = P(X \leq 13) - P(X \leq 7)$</p> <p style="text-align: center;">$= 0.8645 - 0.2202$</p> <p style="text-align: center;">$= 0.6443$ awrt 0.644</p> <p>$X \sim N(10, 7.5)$</p> <p>$P(7.5 \leq X \leq 13.5) = P\left(\frac{7.5-10}{\sqrt{7.5}} \leq X \leq \frac{13.5-10}{\sqrt{7.5}}\right)$</p> <p style="text-align: center;">$= P(-0.913 \leq X \leq 1.278)$</p> <p style="text-align: center;">$= 0.8997 - (1 - 0.8186)$</p> <p style="text-align: center;">$= 0.7183$ awrt 0.718 or 0.719</p> <p>Normal approx /not Poisson since (n is large) and p close to half. or ($np = 10$ $npq = 7.5$) mean \neq variance or $np (= 10)$ and $nq (= 30)$ both > 5. or exact binomial = 0.7148</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1 M1</p> <p>A1 A1</p> <p>M1</p> <p>A1</p> <p>(10)</p> <p>B1</p> <p>B1dep</p> <p>(2)</p>
<p>6a (i)</p> <p>ii)</p>	<p>B1 mean = 10 May be implied in (i) or (ii)</p> <p>M1 Attempting to find $P(X \leq 13) - P(X \leq 7)$</p> <p>A1 awrt 0.644</p> <p>B1 $\sigma^2 = 7.5$ May be implied by being correct in standardised formula</p> <p>M1 using 7.5 or 8.5 or 12.5 or 13.5.</p> <p>M1 standardising using 7.5 or 8 or 8.5 or 12.5 or 13 or 13.5 and their mean and standard deviation.</p>	

b)	<p>A1 award for either $\frac{7.5-10}{\sqrt{7.5}}$ or awrt -0.91</p> <p>A1 award for either $\frac{13.5-10}{\sqrt{7.5}}$ or awrt 1.28</p> <p>M1 Finding the correct area. Following on from their 7.5 and 13.5. Need to do a Prob >0.5 – prob <0.5 or prob <0.5 + prob < 0.5</p> <p>A1 awrt 0.718 or 0.719 only. Dependent on them getting all three method marks.</p> <p>No working but correct answer will gain all the marks</p> <p>first B1 normal</p> <p>second B1 p close to half, or mean \neq variance or np and nq both > 5. They may use a number bigger than 5 or they may work out the exact value 0.7148 using the binomial distribution.</p> <p>Do not allow np > 5 and npq > 5</p>	
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Question Number	Scheme	Marks
2	<p>$X \sim B(100, 0.58)$ $Y \sim N(58, 24.36)$</p> <p>$[P(X > 50) = P(X \geq 51)]$</p> <p style="text-align: right;">using 50.5 or 51.5 or 49.5 or 48.5</p> $= P\left(z \geq \pm \left(\frac{50.5 - 58}{\sqrt{24.36}}\right)\right)$ <p style="text-align: right;">standardising 50.5, 51, 51.5, 48.5, 49, 49.5 and their μ and σ for M1</p> $= P(z \geq -1.52\dots)$ $= 0.9357$ <p><u>alternative</u></p> <p>$X \sim B(100, 0.42)$ $Y \sim N(42, 24.36)$</p> <p>$[P(X < 50) = P(X \leq 49)]$</p> <p style="text-align: right;">using 50.5 or 51.5 or 49.5 or 48.5</p> $= P\left(z \leq \pm \left(\frac{49.5 - 42}{\sqrt{24.36}}\right)\right)$ <p style="text-align: right;">standardising 50.5, 51, 51.5, 48.5, 49, 49.5 and their μ and σ for M1</p> $= P(z \leq 1.52\dots)$ $= 0.9357$	<p>B1 B1 B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;">(7)</p> <p>B1 B1 B1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">(Total 7)</p>
	<p><u>Notes</u></p> <p>The first 3 marks may be given if the following figures are seen in the standardisation formula :- 58 or 42, 24.36 or $\sqrt{24.36}$ or $\sqrt{24.4}$ or awrt 4.94.</p> <p>Otherwise</p> <p>B1 normal</p> <p>B1 58 or 42</p> <p>B1 24.36</p> <p>M1 using 50.5 or 51.5 or 49.5 or 48.5. ignore the direction of the inequality.</p> <p>M1 standardising 50.5, 51, 51.5, 48.5, 49, 49.5 and their μ and σ. They may use $\sqrt{24}$ or $\sqrt{24.36}$ or $\sqrt{24.4}$ or awrt 4.94 for σ or the $\sqrt{\text{of their variance}}$.</p> <p>A1 ± 1.52. may be awarded for $\pm \left(\frac{50.5 - 58}{\sqrt{24.36}}\right)$ or $\pm \left(\frac{49.5 - 42}{\sqrt{24.36}}\right)$ o.e.</p> <p>A1 awrt 0.936</p>	

Question Number	Scheme	Marks
5(a)	$X \sim B(15, 0.5)$	B1 B1 (2)
(b)	$P(X = 8) = P(X \leq 8) - P(X \leq 7) \quad \text{or} \quad \left(\frac{15!}{8!7!} (p)^8 (1-p)^7 \right)$ $= 0.6964 - 0.5$ $= 0.1964$	M1 A1 awrt 0.196 (2)
(c)	$P(X \geq 4) = 1 - P(X \leq 3)$ $= 1 - 0.0176$ $= 0.9824$	M1 A1 (2)
(d)	$H_0 : p = 0.5$ $H_1 : p > 0.5$ $X \sim B(15, 0.5)$ $P(X \geq 13) = 1 - P(X \leq 12)$ $= 1 - 0.9963$ $= 0.0037$ $0.0037 < 0.01$ $[P(X \geq 12) = 1 - 0.9824 = 0.0176] \quad \text{att } P(X \geq 13)$ $P(X \geq 13) = 1 - 0.9963 = 0.0037$ $\text{CR } X \geq 13 \quad \text{awrt } 0.0037 / \text{CR } X \geq 13$ $13 \geq 13$ Reject H_0 or it is significant or a correct statement in context from their values There is sufficient evidence at the 1% significance level that the coin is <u>biased in favour of heads</u> Or There is evidence that Sues belief is correct	B1 B1 M1 A1 M1 A1 (6)
	<u>Notes</u> (a) B1 for Binomial B1 for 15 and 0.5 must be in part a This need not be in the form written (b) M1 attempt to find $P(X = 8)$ any method. Any value of p A1 awrt 0.196 Answer only full marks (c) M1 for $1 - P(X \leq 3)$. A1 awrt 0.982	

- (d) B1 for correct H_0 . must use p or π
B1 for correct H_1 must be one tail must use p or π
M1 attempt to find $P(X \geq 13)$ correctly. E.g. $1 - P(X \leq 12)$
A1 correct probability or CR

To get the next 2 marks the null hypothesis must state or imply that $(p) = 0.5$

M1 for correct statement based on their probability or critical region or a correct contextualised statement that implies that. not just 13 is in the critical region.

A1 This depends on their M1 being awarded for rejecting H_0 . Conclusion in context. Must use the words biased in favour of heads or biased against tails or sues belief is correct .

NB this is a B mark on EPEN.

They may also attempt to find $P(X < 13) = 0.9963$ and compare with 0.99

Question Number	Scheme	Marks
3	<p>(a) $X \sim B(20, 0.3)$</p> <p>$P(X \leq 2) = 0.0355$</p> <p>$P(X \geq 11) = 1 - 0.9829 = 0.0171$</p> <p>Critical region is $(X \leq 2) \cup (X \geq 11)$</p> <p>(b) Significance level = $0.0355 + 0.0171, = 0.0526$ or 5.26%</p> <p>(c) Insufficient evidence to reject H_0 Or sufficient evidence to accept H_0/not significant $x = 3$ (or the value) is not in the critical region or $0.1071 > 0.025$</p> <p>Do not allow inconsistent comments</p>	<p>M1</p> <p>A1 A1 (3)</p> <p>M1 A1 (2)</p> <p>B1 ft</p> <p>B1 ft (2)</p>

Question Number	Scheme	Marks
5	<p>(a) X represents the number of defective components.</p> $P(X = 1) = (0.99)^9 (0.01) \times 10 = 0.0914$ <p>(b) $P(X \geq 2) = 1 - P(X \leq 1)$ $= 1 - (p)^{10} - (a)$ $= 0.0043$</p> <p>(c) $X \sim \text{Po}(2.5)$</p> $P(1 \leq X \leq 4) = P(X \leq 4) - P(X = 0)$ $= 0.8912 - 0.0821$ $= 0.809$ <p>Normal distribution used. B1 for mean only</p> <hr/> <p>Special case for parts a and b If they use 0.1 do not treat as misread as it makes it easier. (a) M1 A0 if they have 0.3874 (b) M1 A1ft A0 they will get 0.2639 (c) Could get B1 B0 M1 A0</p> <hr/> <p>For any other values of p which are in the table do not use misread. Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c) B1 B0 M1 A0</p>	<p>M1A1 (2)</p> <p>M1 A1✓ A1 (3)</p> <p>B1B1 M1 A1 (4)</p>

June 2009
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$[X \sim B(30, 0.15)]$ $P(X \leq 6) = 0.8474$	awrt 0.847 M1, A1 (2)
(b)	$Y \sim B(60, 0.15) \approx Po(9)$ $P(Y \leq 12) = 0.8758$	for using Po(9) B1 awrt 0.876 M1, A1 (3)
[N.B. normal approximation gives 0.897, exact binomial gives 0.894]		[5]
(a)	<p>M1 for a correct probability statement $P(X \leq 6)$ or $P(X < 7)$ or $P(X=0) + P(X=1) + P(X=2) + P(X=4) + P(X=5) + P(X=6)$. (may be implied by long calculation) Correct answer gets M1 A1. allow 84.74%</p>	
(b)	<p>B1 may be implied by using Po(9). Common incorrect answer which implies this is 0.9261 M1 for a correct probability statement $P(X \leq 12)$ or $P(X < 13)$ or $P(X=0) + P(X=1) + \dots + P(X=12)$ (may be implied by long calculation) and attempt to evaluate this probability using their Poisson distribution. Condone $P(X \leq 13) = 0.8758$ for B1 M1 A1 Correct answer gets B1 M1 A1 Use of normal or exact binomial get B0 M0 A0</p>	

Question Number	Scheme	Marks
Q4 (a)	$X \sim B(20, 0.3)$ $P(X \leq 9) = 0.9520$ so Therefore the critical region is $\{X \leq 2\} \cup \{X \geq 10\}$ (b) $0.0355 + 0.0480 = 0.0835$ awrt (0.083 or 0.084) (c) 11 is in the critical region there is evidence of a <u>change/ increase</u> in the <u>proportion/number</u> of <u>customers buying single tins</u>	M1 A1 A1 A1A1 (5) B1 (1) B1ft B1ft (2) [8]
(a)	M1 for B(20,0.3) seen or used 1 st A1 for 0.0355 2 nd A1 for 0.048 3 rd A1 for $(X) \leq 2$ or $(X) < 3$ or $[0,2]$ They get A0 if they write $P(X \leq 2/ X < 3)$ 4 th A1 $(X) \geq 10$ or $(X) > 9$ or $[10,20]$ They get A0 if they write $P(X \geq 10/ X > 9)$ $10 \leq X \leq 2$ etc is accepted To describe the critical regions they can use any letter or no letter at all. It does not have to be X. (b) B1 correct answer only (c) 1 st B1 for a correct statement about 11 and their critical region. 2 nd B1 for a correct comment in context consistent with their CR and the value 11 Alternative solution 1 st B0 $P(X \geq 11) = 1 - 0.9829 = 0.0171$ since no comment about the critical region 2 nd B1 a correct contextual statement.	

Question Number	Scheme	Marks
Q6	<p>(a) The set of values of the test statistic for which the null hypothesis is rejected in a hypothesis test.</p> <p>(b) $X \sim B(30, 0.3)$ $P(X \leq 3) = 0.0093$ $P(X \leq 2) = 0.0021$ $P(X \geq 16) = 1 - 0.9936 = 0.0064$ $P(X \geq 17) = 1 - 0.9979 = 0.0021$ Critical region is $(0 \leq) x \leq 2$ or $16 \leq x (\leq 30)$</p> <p>(c) Actual significance level $0.0021 + 0.0064 = 0.0085$ or 0.85%</p> <p>(d) 15 (it) is not in the critical region not significant No significant evidence of a change in $p = 0.3$ accept H_0, (reject H_1) $P(x \geq 15) = 0.0169$</p>	<p>B1 B1 (2) M1 A1 A1 A1A1 (5) B1 (1) Bft 2, 1, 0 (2) Total [10]</p>
Q6	<p>Notes</p> <p>(a) 1st B1 for “values/ numbers” 2nd B1 for “reject the null hypothesis” o.e or the test is significant</p> <p>(b) M1 for using $B(30, 0.3)$ 1st A1 $P(x \leq 2) = 0.0021$ 2nd A1 0.0064</p> <p>3rd A1 for $(X) \leq 2$ or $(X) < 3$ They get A0 if they write $P(X \leq 2 / X < 3)$ 4th A1 $(X) \geq 16$ or $(X) > 15$ They get A0 if they write $P(X \geq 16 X > 15)$ NB these are B1 B1 but mark as A1 A1</p> <p>$16 \leq X \leq 2$ etc is accepted To describe the critical regions they can use any letter or no letter at all. It does not have to be X.</p> <p>(c) B1 correct answer only</p> <p>(d) Follow through 15 and their critical region B1 for any one of the 5 correct statements up to a maximum of B2 – B1 for any incorrect statements</p>	

Question Number	Scheme	Marks
Q6 (a)	2 outcomes/faulty or not faulty/success or fail A constant probability Independence Fixed number of trials (fixed n)	B1 B1 (2)
(b)	$X \sim B(50, 0.25)$ $P(X \leq 6) = 0.0194$ $P(X \leq 7) = 0.0453$ $P(X \geq 18) = 0.0551$ $P(X \geq 19) = 0.0287$ CR $X \leq 6$ and $X \geq 19$	M1 A1 A1 (3)
(c)	$0.0194 + 0.0287 = 0.0481$	M1A1 (2)
(d)	8(It) is not in the Critical region or 8(It) is not significant or $0.0916 > 0.025$; There is evidence that the probability of a faulty bolt is 0.25 or the company's claim is correct.	M1; A1ft (2)
(e)	$H_0 : p = 0.25$ $H_1 : p < 0.25$ $P(X \leq 5) = 0.0070$ or CR $X \leq 5$ $0.007 < 0.01$, 5 is in the critical region, reject H_0 , significant. There is evidence that the probability of faulty bolts has decreased	B1B1 M1A1 M1 A1ft (6) [15]
	Notes (a) B1 B1 one mark for each of any of the four statements. Give first B1 if only one correct statement given. No context needed. (b) M1 for writing or using $B(50, 0.25)$ also may be implied by both CR being correct. Condone use of P in critical region for the method mark. A1 $(X) \leq 6$ o.e. $[0, 6]$ DO NOT accept $P(X \leq 6)$ A1 $(X) \geq 19$ o.e. $[19, 50]$ DO NOT accept $P(X \geq 19)$ (c) M1 Adding two probabilities for two tails. Both probabilities must be less than 0.5 A1 awrt 0.0481 (d) M1 one of the given statements followed through from their CR. A1 contextual comment followed through from their CR. NB A correct contextual comment alone followed through from their CR. will get M1 A1 (e) B1 for H_0 must use p or π (pi) B1 for H_1 must use p or π (pi) M1 for finding or writing $P(X \leq 5)$ or attempting to find a critical region or a correct critical region A1 awrt 0.007/CR $X \leq 5$ M1 correct statement using their Probability and 0.01 if one tail test or a correct statement using their Probability and 0.005 if two tail test. The 0.01 or 0.005 needn't be explicitly seen but implied by correct statement compatible with their H_1 . If no H_1 given M0 A1 correct contextual statement follow through from their prob and H_1 . Need faulty bolts and decreased. NB A correct contextual statement alone followed through from their prob and H_1 get M1 A1	