January 2008
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
2. (a)	Complete method, using terms of form ar^k, to find r [e.g. **Dividing** $ar^6 = 80$ by $ar^3 = 10$ to find r ; $r^6 - r^3 = 8$ is M0] $r = 2$	M1 A1 (2)
(b)	Complete method for finding a [e.g. Substituting value for r into equation of form $ar^k = 10$ or 80 and finding a value for a.]	M1

	($8a = 10$) $a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)	A1 (2)
(c)	Substituting their values of a and r into **correct** formula for sum. $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this)	M1 A1 (2) **[6]**
Notes:	(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly) (b) M1: Allow for numerical approach: e.g. $\frac{10}{{r_c}^3} \leftarrow \frac{10}{{r_c}^2} \leftarrow \frac{10}{r_c} \leftarrow 10$ In (a) and (b) correct answer, with no working, allow both marks. (c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula **not** quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.	

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6.	(a) $T_{20} = 5\times\left(\frac{4}{5}\right)^{19} = 0.072$ (Accept awrt) Allow $5\times\frac{4}{5}^{19}$ for M1	M1 A1	(2)
	(b) $S_\infty = \frac{5}{1-0.8} = 25$	M1 A1	(2)
	(c) $\frac{5(1-0.8^k)}{1-0.8} > 24.95$ (Allow with = or <)	M1	
	$1-0.8^k > 0.998$ (or equiv., see below) (Allow with = or <)	A1	
	$k\log 0.8 < \log 0.002$ or $k > \log_{0.8} 0.002$ (Allow with = or <)	M1	
	$k > \frac{\log 0.002}{\log 0.8}$ (*)	A1cso	(4)
	(d) $k = 28$ (Must be this integer value) Not $k > 27$, or $k < 28$, or $k > 28$	B1	(1)
			9
	(a) and (b): Correct answer without working scores both marks. (a) M: Requires use of the correct formula ar^{n-1}. (b) M: Requires use of the correct formula $\frac{a}{1-r}$ (c) 1st M: The sum may have already been 'manipulated' (perhaps wrongly), but this mark can still be allowed. 1st A: A 'numerically correct' version that has dealt with $(1-0.8)$ denominator, e.g. $1-\left(\frac{4}{5}\right)^k > 0.998$, $5(1-0.8^k) > 4.99$, $25(1-0.8^k) > 24.95$, $5-5(0.8^k) > 4.99$. In any of these, $\frac{4}{5}$ instead of 0.8 is fine, and condone $\frac{4}{5}^k$ if correctly treated later. 2nd M: Introducing logs and using laws of logs correctly (this must include dealing with the power k so that $p^k = k\log p$). 2nd A: An <u>incorrect</u> statement (including equalities) at any stage in the working loses this mark (this is often identifiable at the stage $k\log 0.8 > \log 0.002$). (So a fully correct method with inequalities is required.)		

Question Number	Scheme	Marks
9 (a)	Initial step: Two of: $a = k+4,\ ar = k,\ ar^2 = 2k-15$ Or one of: $r = \frac{k}{k+4},\quad r = \frac{2k-15}{k},\quad r^2 = \frac{2k-15}{k+4}$, Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$	M1
	$k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$	M1, A1
	Proceed to $k^2 - 7k - 60 = 0$ (*)	A1 (4)
(b)	$(k-12)(k+5) = 0 \qquad k = 12$ (*)	M1 A1 (2)
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k\text{-}15}{k} = \frac{12}{16}\ \left(= \frac{3}{4}\ \text{or}\ 0.75\right)$	M1 A1 (2)
(d)	$\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$	M1 A1 (2) **[10]**
(a) (b) (c) (d)	**M1:** The 'initial step', scoring the first M mark, may be implied by next line of proof **M1**: Eliminates *a* and *r* to give valid equation in *k* only. Can be awarded for equation involving fractions. **A1 :** need some correct expansion and working and answer equivalent to required quadratic but with uncollected terms. Equations involving fractions do not get this mark. (No fractions, no brackets – could be a cubic equation) **A1:** as answer is printed this mark is for cso (Needs = 0) **All four marks must be scored in part (a)** **M1:** Attempt to solve quadratic **A1:** This is for correct factorisation or solution and $k = 12$. Ignore the extra solution ($k = -5$ or even $k = 5$), if seen. Substitute and verify is **M1 A0** **Marks must be scored in part (b)** **M1:** Complete method to find *r* Could have answer in terms of *k* **A1:** 0.75 or any correct equivalent **Both Marks must be scored in (c)** **M1:** Tries to use $\frac{a}{1-r}$, (even with $r>1$). Could have an answer still in terms of *k*. **A1:** This answer is 64 cao.	

Question Number	Scheme	Marks
Q5 (a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$	M1
	$r = \frac{2}{3}$ (*)	A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324$ or $a\left(\frac{2}{3}\right)^5 = 96$ $a = \ldots$, 729	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1-\left[\frac{2}{3}\right]^{15}\right)}{1-\frac{2}{3}}$, $= 2182.00\ldots$ (AWRT 2180)	M1A1ft, (3)
(d)	$S_\infty = \frac{729}{1-\frac{2}{3}}$, $= 2187$	M1, A1 (2)
		[9]

(a)

M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction.

A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp and the final answer 2/3 is seen.

Alternative: (verification)

M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times).

A1 Obtaining 96 (cso). (A conclusion is not required).

$324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.

(b)

M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their *r*) twice from 324 (or 5 times from 96).

Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)… but no other exceptions are allowed.

(c)

M1 for use of sum to 15 terms formula with values of *a* and *r*. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated.

1st A1ft for a correct expression or correct ft their *a* with $r = \frac{2}{3}$.

2nd A1 for awrt 2180, even following 'minor inaccuracies'.

Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c).

Alternative:

M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their *a* and $r = \frac{2}{3}$.

(d)

M1 for use of correct sum to infinity formula with their *a*. For this mark, if a value of *r* different from the given value is being used, M1 can still be allowed providing $|r| < 1$.

Question Number	Scheme	Marks
Q6 (a)	$18000\times(0.8)^3 \quad = £9216 *$ [may see $\frac{4}{5}$ or 80% or equivalent].	B1cso (1)
(b)	$18000\times(0.8)^n < 1000$	M1
	$n\log(0.8) < \log\left(\frac{1}{18}\right)$	M1
	$n > \dfrac{\log\left(\frac{1}{18}\right)}{\log(0.8)} = 12.952....$ so n = 13.	A1 cso (3)
(c)	$u_5 = 200\times(1.12)^4$, = £314.70 or £314.71	M1, A1 (2)
(d)	$S_{15} = \dfrac{200\left(1.12^{15}-1\right)}{1.12-1}$ or $\dfrac{200\left(1-1.12^{15}\right)}{1-1.12}$, = 7455.94….. awrt £7460	M1A1, A1 (3) **[9]**
(a)	B1 NB Answer is printed **so need working**. May see as above or $\times 0.8$ in three steps giving 14400, 11520, 9216. Do not need to see £ sign but should see 9216 .	
(b)	1st M1 for an attempt to use *n*th term and 1000. Allow *n* or *n* – 1 and allow > or = 2nd M1 for use of logs to find *n* Allow *n* or *n* – 1 and allow > or = A1 Need *n* = 13 This is an accuracy mark and must follow award of both M marks but should not follow incorrect work using *n* – 1 for example. Condone slips in inequality signs here.	
(c)	M1 for use of their *a* and *r* in formula for 5th term of GP A1 cao need one of these answers – answer can imply method here NB 314.7 – A0	
(d)	M1 for use of sum to 15 terms of GP using their *a* and their *r* (allow if formula stated correctly and one error in substitution, but must use *n* not *n* - 1) 1st A1 for a fully correct expression (not evaluated)	
(b)	Alternative Methods Trial and Improvement See 989.56 (or 989 or 990) identified with 12, 13 or 14 years for **first M1** See 1236.95 (or 1236 or 1237) identified with 11, 12 or 13 years for second **M1** Then *n* = 13 is **A1 (needs both Ms)** **Special case** $18000\times(0.8)^n < 1000$ so *n* = 13 as 989.56<1000 is M1M0A0 (not discounted *n* = 12)	
(c)	May see the terms 224, 250.88, 280.99, 314.71 with a small slip for M1 A0, or done accurately for M1A1	
(d)	Adds 15 terms 200 + 224 + 250.88+… + (977.42) **M1** Seeing 977… is **A1** Obtains answer 7455.94 **A1** or awrt £7460 NOT 7450	

Question Number	Scheme	Marks
9	(a) $25\,000 \times 1.03 = 25750$ $\left\{25000+750=25750, \text{ or } 25000\frac{(1-0.03^2)}{1-0.03}=25750\right\}$ (*)	B1 (1)
	(b) r = 1.03 Allow $\frac{103}{100}$ or $1\frac{3}{100}$ but no other alternatives	B1 (1)
	(c) $25000r^{N-1} > 40000$ (Either letter r or their r value) Allow ' = ' or '<' $r^M > 1.6 \Rightarrow \log r^M > \log 1.6$ Allow ' = ' or '<' (<u>See below</u>) OR (by change of base), $\log_{1.03} 1.6 < M \Rightarrow \frac{\log 1.6}{\log 1.03} < M$ $(N-1)\log 1.03 > \log 1.6$ (Correct bracketing required) (*) Accept work for part (c) seen in part (d)	M1 M1 A1 cso (3)
	(d) Attempt to evaluate $\frac{\log 1.6}{\log 1.03}+1$ {or $25000(1.03)^{15}$ <u>and</u> $25000(1.03)^{16}$ } N = 17 (<u>not</u> 16.9 and not e.g. $N \geq 17$) Allow '17th year' Accept work for part (d) seen in part (c)	M1 A1 (2)
	(e) Using formula $\frac{a(1-r^n)}{1-r}$ with values of a and r, and n = 9, 10 or 11 $\frac{25000(1-1.03^{10})}{1-1.03}$ 287 000 (<u>must</u> be rounded to the nearest 1 000) Allow 287000.00	M1 A1 A1 (3) **10**

(c) 2nd M: Requires $\frac{40000}{25000}$ to be dealt with, and 'two' logs introduced.

With, say, N instead of $N-1$, this mark is still available.

> <u>Jumping</u> straight from $1.03^{N-1} > 1.6$ to $(N-1)\log 1.03 > \log 1.6$ can score only M1 M0 A0.
> (The intermediate step $\log 1.03^{N-1} > \log 1.6$ must be seen).

<u>Longer methods</u> require correct log work throughout for 2nd M, e.g.:

$\log(25000r^{N-1}) > \log 40000 \Rightarrow \log 25000 + \log r^{N-1} > \log 40000 \Rightarrow$

$\log r^{N-1} > \log 40000 - \log 25000 \Rightarrow \log r^{N-1} > \log 1.6$

(d) Correct answer with no working scores both marks.

Evaluating $\log\left(\frac{1.6}{1.03}\right)+1$ does <u>not</u> score the M mark.

(e) M1 can also be scored by a "year by year" method, <u>with terms added</u>.
(Allow the M mark if there is evidence of adding 9, 10 or 11 terms).
1st A1 is scored if the 10 correct terms have been added (allow terms to be to the nearest 100).
To the nearest 100, these terms are:
25000, 25800, 26500, 27300, 28100, 29000, 29900, 30700, 31700, 32600

<u>No</u> working shown: Special case: 287 000 scores 1 mark, scored on ePEN as 1, 0, 0.
(Other answers with no working score no marks).