Question Number	Scheme	Marks
7.	(a) $\dfrac{dy}{dx} = 6\cos 2x - 8\sin 2x$	M1 A1
	$\left(\dfrac{dy}{dx}\right)_0 = 6$	B1
	$y - 4 = -\dfrac{1}{6}x$ or equivalent	M1 A1 (5)
	(b) $R = \sqrt{\left(3^2 + 4^2\right)} = 5$	M1 A1
	$\tan\alpha = \dfrac{4}{3}$, $\alpha \approx 0.927$ awrt 0.927	M1 A1 (4)
	(c) $\sin\left(2x + \text{ their } \alpha\right) = 0$	M1
	$x = -2.03, -0.46, 1.11, 2.68$	A1 A1 A1 (4)
	First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.	**[13]**

Question Number	Scheme	Marks
2.	(a) $R^2 = 5^2 + 12^2$	M1
	$R = 13$	A1
	$\tan\alpha = \frac{12}{5}$	M1
	$\alpha \approx 1.176$ cao	A1 (4)
	(b) $\cos(x-\alpha) = \frac{6}{13}$	M1
	$x - \alpha = \arccos\frac{6}{13} = 1.091\ \ldots$	A1
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$ awrt 2.3	A1
	$x - \alpha = -1.091\ \ldots$ accept … = 5.19 … for M	M1
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$ awrt 0.084 or 0.085	A1 (5)
	(c)(i) $R_{\max} = 13$ ft their R	B1 ft
	(ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x - \alpha = 0$	M1
	$x = \alpha = 1.176\ \ldots$ awrt 1.2, ft their α	A1ft (3)
		[12]

Question Number	Scheme		Marks	
6.	(a)(i) $\sin 3\theta = \sin(2\theta+\theta)$			
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1-2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta$ $\ast$	cso	A1	**(4)**
	(ii) $8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$			
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$		A1 A1	**(5)**
	(b) $\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$		M1	
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ $\ast$	cso	A1	**(4)**
				[13]
	Alternatives to (b)			
	① $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$		M1	
	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ $\ast$	cso	A1	**(4)**
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$			
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$			
	$\sin^2 15° = \frac{2-\sqrt{3}}{4}$		M1 A1	
	$\left(\frac{1}{4}(\sqrt{6}-\sqrt{2})\right)^2 = \frac{1}{16}(6+2-2\sqrt{12}) = \frac{2-\sqrt{3}}{4}$		M1	
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ $\ast$	cso	A1	**(4)**

Question Number	Scheme		Marks
Q6 (a)	$A = B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives		
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	Complete proof, with a link between LHS and RHS. No errors seen.	A1 AG
			(2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\dfrac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\dfrac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\dfrac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$		
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$		
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 AG
			(3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$		
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$		
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2 + 4^2}\,;= \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$	M1
		awrt 36.87	A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$		
			(3)

Question Number	Scheme		Marks
(d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^{\circ}$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^{\circ}$		
	Hence, $x = 51.64591...^{\circ},\ 165.22409...^{\circ}$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1
		Both awrt 51.6 AND awrt 165.2	A1
			(4)
		If there are any EXTRA solutions inside the range $0 \leq x < 180^{\circ}$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \leq x < 180^{\circ}$.	
			[12]

Question Number	Scheme		Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x+\alpha),\quad R>0,\ 0<x<\frac{\pi}{2}$		
	$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$		
	$R = \sqrt{5^2+3^2}\ ; = \sqrt{34}\ \{= 5.83095..\}$	$R^2 = 5^2 + 3^2$ $\sqrt{34}$ or awrt 5.8	M1; A1
	$\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$	$\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$	M1
		$\alpha =$ awrt 0.54 or $\alpha =$ awrt 0.17π or $\alpha = \dfrac{\pi}{\text{awrt } 5.8}$	A1
	Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$		
			(4)
(b)	$5\cos x - 3\sin x = 4$		
	$\sqrt{34}\cos(x+0.5404) = 4$		
	$\cos(x+0.5404) = \dfrac{4}{\sqrt{34}}\ \{= 0.68599...\}$	$\cos(x \pm \text{their } \alpha) = \dfrac{4}{\text{their } R}$	M1
	$(x+0.5404) = 0.814826916...^c$	For applying $\cos^{-1}\left(\dfrac{4}{\text{their } R}\right)$	M1
	$x = 0.2744...^c$	awrt 0.27^c	A1
	$(x+0.5404) = 2\pi - 0.814826916...^c\ \{= 5.468358...^c\}$	$2\pi -$ their 0.8148	ddM1
	$x = 4.9279...^c$	awrt 4.93^c	A1
	Hence, $x = \{0.27, 4.93\}$		
			(5)
			[9]

Part (b): If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

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Question Number	Scheme	Marks
7. (a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$	M1A1
		(3)
(b) (i)	Max Value = 2.5	B1 $\sqrt{\ }$
(ii)	$\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their } \alpha = \frac{\pi}{2}}$; $\Rightarrow \theta = \text{awrt } 2.21$	M1;A1 $\sqrt{\ }$
		(3)
(c)	$H_{\text{Max}} = 8.5$ (m)	B1 $\sqrt{\ }$
	$\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{ their (b) answer}}$; $\Rightarrow t = \text{awrt } 4.41$	M1;A1
		(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1
	Either $t = \text{awrt } 2.1$ or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^{c}\}$	ddM1
	Times = $\{14:06, 18:43\}$	A1 (6)
		[15]
	(a) B1: $R = 2.5$ or $R = \sqrt{6.25}$. For $R = \pm 2.5$, award B0. M1: $\tan\alpha = \pm\frac{1.5}{2}$ or $\tan\alpha = \pm\frac{2}{1.5}$ A1: $\alpha = \text{awrt } 0.6435$ (b) B1 $\sqrt{\ }$: 2.5 or follow through the value of R in part (a). M1: For $\sin(\theta - \text{their } \alpha) = 1$ A1 $\sqrt{\ }$: awrt 2.21 or $\frac{\pi}{2}$ + their α rounding correctly to 3 sf. (c) B1 $\sqrt{\ }$: 8.5 or 6 + their R found in part (a) as long as the answer is greater than 6. M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1$ or $\frac{4\pi t}{25} = $ their (b) answer A1: For $\sin^{-1}(0.4)$ This can be implied by awrt 4.41 or awrt 4.40. (d) M1: $6 + (\text{their } R)\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7$, M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}$ A1: For $\sin^{-1}(0.4)$. This can be implied by awrt 0.41 or awrt 2.73 or other values for different α's. Note this mark can be implied by seeing 1.055. A1: Either $t = \text{awrt } 2.1$ or $t = \text{awrt } 6.7$ ddM1: either π – their PV^{c}. Note that this mark is dependent upon the two M marks. This mark will usually be awarded for seeing either 2.730… or 3.373… A1: Both $t = 14:06$ and $t = 18:43$ or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min.	