Question Number	Scheme		Marks
7. (a)	$\left[x=\ln(t+2),\ y=\frac{1}{t+1}\right],\quad \Rightarrow \frac{dx}{dt}=\frac{1}{t+2}$	Must state $\frac{dx}{dt}=\frac{1}{t+2}$	B1
	$\text{Area}(R)=\int_{\ln 2}^{\ln 4}\frac{1}{t+1}dx;=\int_0^2\left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$	Area $=\int\frac{1}{t+1}dx$. Ignore limits.	M1;
		$\int\left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right)dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x=\ln 2\ \Rightarrow \ln 2=\ln(t+2)\ \Rightarrow\ 2=t+2\ \Rightarrow t=0$ $x=\ln 4\ \Rightarrow \ln 4=\ln(t+2)\ \Rightarrow\ 4=t+2\ \Rightarrow t=2$	changes limits $x\to t$ so that $\ln 2\to 0$ and $\ln 4\to 2$	B1
	Hence, $\text{Area}(R)=\int_0^2\frac{1}{(t+1)(t+2)}dt$		**[4]**
(b)	$\left(\frac{1}{(t+1)(t+2)}\right)=\frac{A}{(t+1)}+\frac{B}{(t+2)}$	$\frac{A}{(t+1)}+\frac{B}{(t+2)}$ with A and B found	M1
	$1=A(t+2)+B(t+1)$		
	Let $t=-1,\quad 1=A(1)\quad\Rightarrow\ \underline{A=1}$ Let $t=-2,\quad 1=B(-1)\quad\Rightarrow\ \underline{B=-1}$	Finds both A and B correctly. Can be implied. (See note below)	A1
	$\int_0^2\frac{1}{(t+1)(t+2)}dt=\int_0^2\frac{1}{(t+1)}-\frac{1}{(t+2)}dt$		
	$=\left[\ln(t+1)-\ln(t+2)\right]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$ Both ln terms correctly ft.	dM1 A1 $\sqrt{}$
	$=(\ln 3-\ln 4)-(\ln 1-\ln 2)$	Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round.	ddM1
	$=\ln 3-\ln 4+\ln 2=\ln 3-\ln 2=\ln\left(\frac{3}{2}\right)$	$\underline{\ln 3-\ln 4+\ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right)-\ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3-\ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1)	A1 aef isw **[6]**

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)}=\frac{1}{(t+1)}+\frac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\frac{1}{(t+1)(t+2)}=\frac{1}{(t+1)}-\frac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme		Marks
	$x = \ln(t+2)$, $\quad y = \dfrac{1}{t+1}$		
7. (c)	$e^x = t+2 \;\Rightarrow\; t = e^x - 2$	Attempt to make $t =\ldots$ the subject giving $t = e^x - 2$	M1 A1
	$y = \dfrac{1}{e^x - 2 + 1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \dfrac{1}{e^x - 1}$	dM1 A1 **[4]**
Aliter **7.** (c) **Way 2**	$t+1 = \dfrac{1}{y} \;\Rightarrow\; t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$ $y(t+1) = 1 \;\Rightarrow\; yt + y = 1 \;\Rightarrow yt = 1 - y \;\Rightarrow t = \dfrac{1-y}{y}$	Attempt to make $t =\ldots$ the subject Giving either $t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$	M1 A1
	$x = \ln\left(\dfrac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\dfrac{1-y}{y} + 2\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$		
	$e^x = \dfrac{1}{y} + 1$		
	$e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1 **[4]**
(d)	Domain : $\underline{x > 0}$	$\underline{x > 0}$ or just > 0	B1 **[1]**
			15 marks

Question Number	Scheme		Marks
***Aliter* 7. (c) Way 3**	$e^x = t+2 \;\Rightarrow\; t+1 = e^x - 1$	Attempt to make $t+1 = \ldots$ the subject giving $t+1 = e^x - 1$	M1 A1
	$y = \dfrac{1}{t+1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \dfrac{1}{e^x - 1}$	dM1 A1 **[4]**
***Aliter* 7. (c) Way 4**	$t+1 = \dfrac{1}{y} \;\Rightarrow\; t+2 = \dfrac{1}{y} + 1$ or $t+2 = \dfrac{1+y}{y}$	Attempt to make $t+2 = \ldots$ the subject Either $t+2 = \dfrac{1}{y} + 1$ or $t+2 = \dfrac{1+y}{y}$	M1 A1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$		
	$e^x = \dfrac{1}{y} + 1 \;\Rightarrow\; e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1 **[4]**

Question Number	Scheme		Marks
8. (a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	$\underline{t = \frac{\pi}{3}}$ or $\underline{\text{awrt } 1.05}$ (radians) only stated in the range $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
			[2]
(b)	$x = 8\cos t$, $y = 4\sin 2t$		
	$\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{ = \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence m(**N**) = $-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses m(**N**) $= -\frac{1}{\text{their m}(\mathbf{T})}$.	dM1*
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their m}_N)x + "c"$.	dM1*
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so **N**: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Question	Scheme		Marks
8. (c)	$A = \int_0^4 y\,dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,dt$	attempt at $A = \int y \frac{dx}{dt}\,dt$ correct expression (ignore limits and dt)	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,dt$	Seeing $\sin 2t = 2\sin t\cos t$ anywhere in PART (c).	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,dt$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,dt$	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**
			[4]
(d)	{Using substitution $u = \sin t \Rightarrow \frac{du}{dt} = \cos t$} {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$}		
	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	$k\sin^3 t$ or ku^3 with $u = \sin t$ Correct integration ignoring limits.	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	Substitutes limits of either $(t = \frac{\pi}{2}$ and $t = \frac{\pi}{3})$ or $(u = 1$ and $u = \frac{\sqrt{3}}{2})$ and subtracts the correct way round.	dM1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	$\underline{\frac{64}{3} - 8\sqrt{3}}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.	A1 aef **isw**
	(Note that $a = \frac{64}{3}$, $b = -8$)		**[4]**
			16 marks

Question Number	Scheme		Marks
7. (a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 **[1]**
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{\mathrm{d}x}{\mathrm{d}t}=3t^2-8$, $\frac{\mathrm{d}y}{\mathrm{d}t}=2t$		
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2t}{3t^2-8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ Correct $\frac{dy}{dx}$	M1 A1
	At A, $\mathrm{m}(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	<u>A1</u>
	$\mathbf{T}$: $y-(\text{their } 1)=m_T\left(x-(\text{their } 7)\right)$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$ Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 **cso** **[5]**
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}=55.125$ or awrt 55.1 $y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3 Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	Candidate uses their value of t to find either the x or y coordinate One of either x or y correct. Both x and y correct. awrt	ddM1 A1 A1 **[6]**
			12 marks

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
  Oe or equivalent.

Question 7

Appendix to Jan 2009 Mark Scheme

Question Number	Scheme		Marks
7. (a)	It is acceptable for a candidate to write $x = 7,\ y = 1,$ to gain B1.	$A(7,1)$	B1
			[1]
Aliter **(c) Way 2**	$x = t^3 - 8t = t(t^2 - 8) = t(y-8)$		
	So, $x^2 = t^2(y-8)^2 = y(y-8)^2$		
	$2x - 5y - 9 = 0 \Rightarrow 2x = 5y + 9 \Rightarrow 4x^2 = (5y+9)^2$		
	Hence, $4y(y-8)^2 = (5y+9)^2$	Forming an equation in terms of y only.	M1
	$4y(y^2 - 16y + 64) = 25y^2 + 90y + 81$		
	$4y^3 - 64y^2 + 256y = 25y^2 + 90y + 81$		
	$4y^3 - 89y^2 + 166y - 81 = 0$		
	$(y-1)(y-1)(4y-81) = 0$	A realisation that $(y-1)$ is a factor.	dM1
		Correct factorisation	A1
	$y = \frac{81}{4} = 20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x^2 = \frac{81}{4}(\frac{81}{4} - 8)^2$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x = \frac{441}{8} = 55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$		**[6]**

Question Number	Scheme		Marks
Aliter 7. (c) **Way 3**	$t = \sqrt{y}$		
	So $x = \left(\sqrt{y}\right)^3 - 8\left(\sqrt{y}\right)$		
	$2x - 5y - 9 = 0$ yields		
	$2\left(\sqrt{y}\right)^3 - 16\left(\sqrt{y}\right) - 5y - 9 = 0$	Forming an equation in terms of y only.	M1
	$\Rightarrow 2\left(\sqrt{y}\right)^3 - 5y - 16\left(\sqrt{y}\right) - 9 = 0$		
	$\left(\sqrt{y}+1\right)\left\{\left(2y - 7\sqrt{y} - 9\right) = 0\right\}$	A realisation that $\left(\sqrt{y}+1\right)$ is a factor.	dM1
	$\left(\sqrt{y}+1\right)\left\{\left(\sqrt{y}+1\right)\left(2\sqrt{y} - 9\right) = 0\right\}$	Correct factorisation.	A1
	$y = \frac{81}{4} = 20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x = \left(\sqrt{\frac{81}{4}}\right)^3 - 8\left(\sqrt{\frac{81}{4}}\right)$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x = \frac{441}{8} = 55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$		**[6]**

Question Number	Scheme	Marks
Q8 (a)	$\int \sin^2\theta \, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$	
	$\pi\int y^2\, dx = \pi\int y^2 \frac{dx}{d\theta}\, d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta \, d\theta$	M1 A1
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}\, d\theta$	M1
	$= 16\pi\int \sin^2\theta\, d\theta \qquad k = 16\pi$	A1
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta\, d\theta\right)$	
(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ Use of correct limits	M1
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3} \qquad p = \frac{4}{3},\ q = -2$	A1 (3)
		[10]

Question Number	Scheme		Marks	
Q7	(a) $y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$			
	$t=0,3,-3$	Any one correct value	B1	
	At $t=0$, $x=5(0)^2-4=-4$	Method for finding one value of x	M1	
	At $t=3$, $x=5(3)^2-4=41$			
	$\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$			
	At A, $x=-4$; at B, $x=41$	Both	A1	(3)
	(b) $\dfrac{dx}{dt}=10t$	Seen or implied	B1	
	$\int y\,dx=\int y\frac{dx}{dt}\,dt=\int t(9-t^2)10t\,dt$		M1 A1	
	$=\int(90t^2-10t^4)\,dt$			
	$=\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\ (+C)\qquad \left(=30t^3-2t^5\ (+C)\right)$		A1	
	$\left[\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$		M1	
	$A=2\int y\,dx=648\quad(\text{units}^2)$		A1	(6)
				[9]

Question Number	Scheme		Marks
4.	(a) $\frac{dx}{dt} = 2\sin t\cos t,\ \frac{dy}{dt} = 2\sec^2 t$		B1 B1
	$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t\cos t}\left(= \frac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1 **(4)**
	(b) At $t = \frac{\pi}{3},\ x = \frac{3}{4},\ y = 2\sqrt{3}$		B1
	$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \frac{16}{\sqrt{3}}$		M1 A1
	$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$		M1
	$y = 0 \quad \Rightarrow \quad x = \frac{3}{8}$		M1 A1 **(6)**
			[10]