Jan 08

Question Number	Scheme		Marks	
3.	(a) $\mathrm{f}(2) = 0.38 \ \ldots$			
	$\mathrm{f}(3) = -0.39 \ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ $\ast$	cso	A1	(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$		M1	
	$x_2 \approx 2.50498$		A1	
	$x_3 \approx 2.50518$		A1	(3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.		M1	
	$\mathrm{f}(2.5045) \approx 6 \times 10^{-4}$			
	$\mathrm{f}(2.5055) \approx -2 \times 10^{-4}$			
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$			
	$\Rightarrow$ root $= 2.505$ to 3 dp $\ast$	cso	A1	(2)
				[7]
	Note: The root, correct to 5 dp, is 2.50524			

Jun 08

Question Number	Scheme		Marks	
7.	(a) $\mathrm{f}(1.4) = -0.568\ \ldots < 0$			
	$\mathrm{f}(1.45) = 0.245\ \ldots > 0$		M1	
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4, 1.45)$		A1	(2)
	(b) $3x^3 = 2x + 6$			
	$x^3 = \frac{2x}{3} + 2$			
	$x^2 = \frac{2}{3} + \frac{2}{x}$		M1 A1	
	$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ ✱	cso	A1	(3)
	(c) $x_1 = 1.4371$		B1	
	$x_2 = 1.4347$		B1	
	$x_3 = 1.4355$		B1	(3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.		M1	
	$\mathrm{f}(1.4345) = -0.01$ ...			
	$\mathrm{f}(1.4355) = 0.003\ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4345, 1.4355)$			
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places ✱	cso	A1	(3) **[11]**
	Note: $\alpha = 1.435\,304\,553\ \ldots$			

Jan 09

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}'(x) = 3\mathrm{e}^x + 3x\mathrm{e}^x$	M1 A1
	$3\mathrm{e}^x + 3x\mathrm{e}^x = 3\mathrm{e}^x(1+x) = 0$	
	$x = -1$	M1 A1
	$\mathrm{f}(-1) = -3\mathrm{e}^{-1} - 1$	B1 **(5)**
	(b) $x_1 = 0.2596$	B1
	$x_2 = 0.2571$	B1
	$x_3 = 0.2578$	B1 **(3)**
	(c) Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$\mathrm{f}(0.257\,55) = -0.000\,379\ ...$	
	$\mathrm{f}(0.257\,65) = 0.000\,109\ \ldots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ∗ cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	**(3)** **[11]**
	Note: $x = 0.257\,627\,65\ \ldots$ is accurate	

June 2009
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme		Marks
Q1 (a)	Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \frac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 cso
			(3)
(b)	Let $f(x) = -x^3 + 2x^2 + 2 = 0$		
	$f(2.3585) = 0.00583577...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
	$f(2.3595) = -0.00142286...$	any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

Question Number	Scheme		Marks
Q2	$\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$\mathrm{f}(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$	Sets $\mathrm{f}(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \dfrac{3x+11}{x+2}$ $\Rightarrow x = \sqrt{\left(\dfrac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 **AG**
			(2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\dfrac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\dfrac{3(0)+11}{(0)+2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$	Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037	A1
	$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1
			(3)
(c)	Let $\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11 = 0$		
	$\mathrm{f}(2.0565) = -0.013781637...$ $\mathrm{f}(2.0575) = 0.0041401094...$	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter	M1
		any one value awrt 1 sf	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	both values correct awrt 1sf, sign change and conclusion	A1
			(3)
		As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	
			[8]

Question Number	Scheme	Marks
3. (a)	$\mathrm{f}(1.2) = 0.49166551...,\quad \mathrm{f}(1.3) = -0.048719817...$ Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1 (2)
(b)	$4\operatorname{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\operatorname{cosec}x + 1$	M1
	$\Rightarrow x = \operatorname{cosec}x + \frac{1}{4} \Rightarrow \underline{x = \frac{1}{\sin x} + \frac{1}{4}}$	A1 * (2)
(c)	$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$	M1
	$x_1 = 1.303757858...,\quad x_2 = 1.286745793...$	A1
	$x_3 = 1.291744613...$	A1 (3)
(d)	$\mathrm{f}(1.2905) = 0.00044566695...,\quad \mathrm{f}(1.2915) = -0.00475017278...$	M1
	Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	A1 (2) **[9]**
	(a) M1: Attempts to evaluate both $\mathrm{f}(1.2)$ and $\mathrm{f}(1.3)$ and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $\mathrm{f}(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by $x_1 =$ awrt 1.3 or $x_1 =$ awrt 46°. A1: Both $x_1 =$ awrt 1.3038 and $x_2 =$ awrt 1.2867 A1: $x_3 =$ awrt 1.2917 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x). A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	