Question Number	Scheme		Marks
5. (a)	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \frac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y=16}$ and $\underline{y=8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{ \frac{\cancel{d}y}{\cancel{d}x} \times \right\}$ $3x^2 - 8y\frac{dy}{dx}; = \left(\underline{12y + 12x\frac{dy}{dx}} \right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = ...$ Correct LHS equation; Correct application of product rule	M1 A1; (B1)
	$\left\{ \frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y} \right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$, @ $(-8, 16)$, $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found.	dM1 A1 A1 **cso**
			[6]
			9 marks

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 2**	$\left\{\frac{\cancel{dx}}{\cancel{dy}}\cancel{=}\right\}\quad 3x^2\frac{dx}{dy}-8y;\ =\underline{\left(12y\frac{dx}{dy}+12x\right)}$	Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$. Ignore $\frac{dx}{dy}=\ldots$	M1
		Correct LHS equation	A1;
		<u>Correct application of product rule</u>	(B1)
	$\left\{\frac{dy}{dx}=\frac{3x^2-12y}{12x+8y}\right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{dy}{dx}=\frac{3(64)-12(8)}{12(-8)+8(8)}=\frac{96}{-32}=\underline{-3}$,	Substitutes $x=-8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$.	dM1
	@ $(-8, 16)$, $\frac{dy}{dx}=\frac{3(64)-12(16)}{12(-8)+8(16)}=\frac{0}{32}=\underline{0}$.	One gradient found.	A1
		Both gradients of <u>-3</u> and <u>0</u> ***correctly*** found.	A1 **cso**
			[6]

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 3**	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$4y^2 + 12xy - x^3 = 0$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$		
	$y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$		
	$y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$		
	$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}}; \left(18x + 3x^2\right)$	A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.	M1
	$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$	$\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$	A1
		$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}}; \left(18x + 3x^2\right)$	A1
	@ $x = -8$ $\quad \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$	Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1
	$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$		
	$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3}, \underline{0}.$	One gradient correctly found.	A1
		Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.	A1
			[6]

Question Number	Scheme		Marks
4. (a)	$3x^2 - y^2 + xy = 4$ (eqn $*$)		
	$\left\{\frac{dy}{dx} \times\right\}$ $\underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} =\right)$)	M1
		Correct application ( ) of product rule	B1
		$(3x^2 - y^2) \to \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \to \underline{0})$	A1
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1$*$
	giving $-18x - 3y = 8x - 16y$		
	giving $13y = 26x$	Attempt to combine either terms in x or terms in y together to give either ax or by.	dM1$*$
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**
			[6]
(b)	At P & Q, $y = 2x$. Substituting into eqn $*$		
	gives $3x^2 - (2x)^2 + x(2x) = 4$	Attempt replacing y by $2x$ **in at least one** of the y terms in eqn$*$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	Either $x = 2$ or $x = -2$	A1
	$y = 2x \Rightarrow y = \pm 4$		
	Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	Both $\underline{(2,4)}$ and $\underline{(-2,-4)}$	A1
			[3]
			9 marks

Question Number	Scheme		Marks
1. (a)	$C:\ y^2 - 3y = x^3 + 8$		
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \times\right\}\quad 2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{dy}{dx} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their ' $2y\frac{dy}{dx} - 3\frac{dy}{dx}$ '. Can be implied.	M1
	$\frac{dy}{dx} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			[4]
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2,3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$	$\frac{dy}{dx} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			[3]
			7 marks

1(b) final A1 $\sqrt{}$ **.** Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} =$ their x^2, ***may*** indicate a correct follow through.

Question Number	Scheme		Marks	
Q4 (a)	$e^{-2x}\dfrac{dy}{dx}-2ye^{-2x}=2+2y\dfrac{dy}{dx}$	A1 correct RHS	M1 A1	
	$\dfrac{d}{dx}\left(ye^{-2x}\right)=e^{-2x}\dfrac{dy}{dx}-2ye^{-2x}$		B1	
	$\left(e^{-2x}-2y\right)\dfrac{dy}{dx}=2+2ye^{-2x}$		M1	
	$\dfrac{dy}{dx}=\dfrac{2+2ye^{-2x}}{e^{-2x}-2y}$		A1	(5)
(b)	At P , $\dfrac{dy}{dx}=\dfrac{2+2e^{0}}{e^{0}-2}=-4$		M1	
	Using $mm'=-1$			
	$m'=\dfrac{1}{4}$		M1	
	$y-1=\dfrac{1}{4}(x-0)$		M1	
	$x-4y+4=0$	or any integer multiple	A1	(4)
				[9]
	Alternative for (a) *differentiating implicitly with respect to y.*			
	$e^{-2x}-2ye^{-2x}\dfrac{dx}{dy}=2\dfrac{dx}{dy}+2y$	A1 correct RHS	M1 A1	
	$\dfrac{d}{dy}\left(ye^{-2x}\right)=e^{-2x}-2ye^{-2x}\dfrac{dx}{dy}$		B1	
	$\left(2+2ye^{-2x}\right)\dfrac{dx}{dy}=e^{-2x}-2y$		M1	
	$\dfrac{dx}{dy}=\dfrac{e^{-2x}-2y}{2+2ye^{-2x}}$			
	$\dfrac{dy}{dx}=\dfrac{2+2ye^{-2x}}{e^{-2x}-2y}$		A1	(5)

Question Number	Scheme	Marks
Q3	(a) $-2\sin 2x - 3\sin 3y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2\sin 2x}{3\sin 3y}$ Accept $\dfrac{2\sin 2x}{-3\sin 3y}, \dfrac{-2\sin 2x}{3\sin 3y}$	A1 (3)
	(b) At $x = \dfrac{\pi}{6}$, $\cos\left(\dfrac{2\pi}{6}\right) + \cos 3y = 1$	M1
	$\cos 3y = \dfrac{1}{2}$	A1
	$3y = \dfrac{\pi}{3} \Rightarrow y = \dfrac{\pi}{9}$ awrt 0.349	A1 (3)
	(c) At $\left(\dfrac{\pi}{6}, \dfrac{\pi}{9}\right)$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\dfrac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\dfrac{2}{3}$	M1
	$y - \dfrac{\pi}{9} = -\dfrac{2}{3}\left(x - \dfrac{\pi}{6}\right)$	M1
	Leading to $6x + 9y - 2\pi = 0$	A1 (3) **[9]**

Question Number	Scheme		Marks
3.	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2^x\right) = \ln 2 . 2^x$		B1
	$\ln 2 . 2^x + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y + 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}$		M1 A1= A1
	Substituting $(3, 2)$		
	$8\ln 2 + 4\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4 + 6\dfrac{\mathrm{d}y}{\mathrm{d}x}$		M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4\ln 2 - 2$	Accept exact equivalents	M1 A1 **(7)**
			[7]