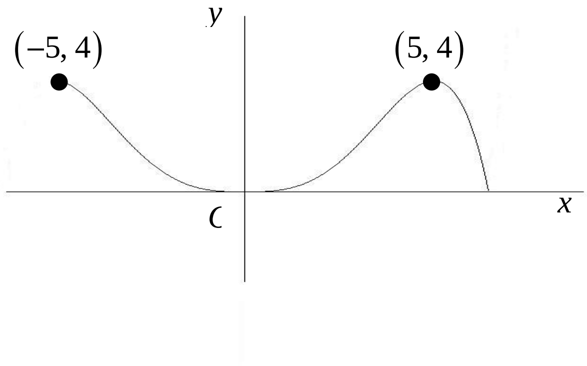
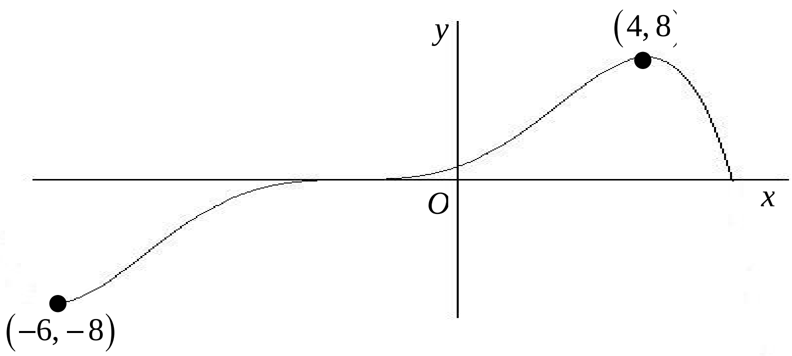
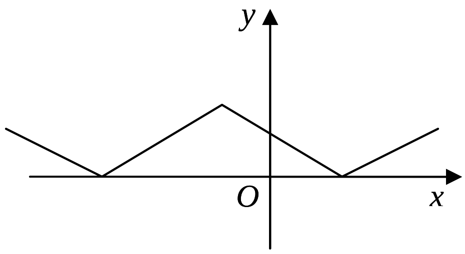
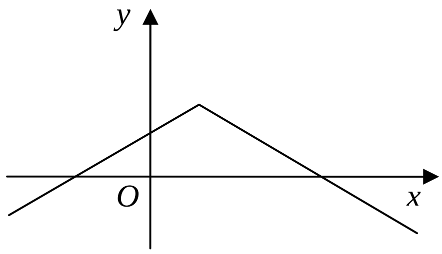
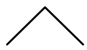
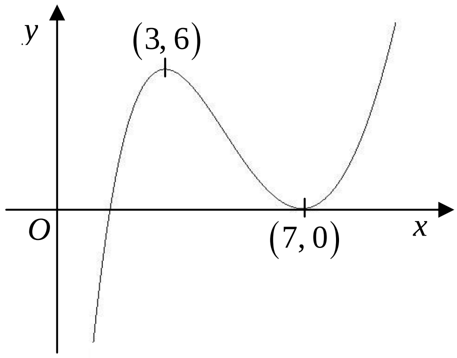
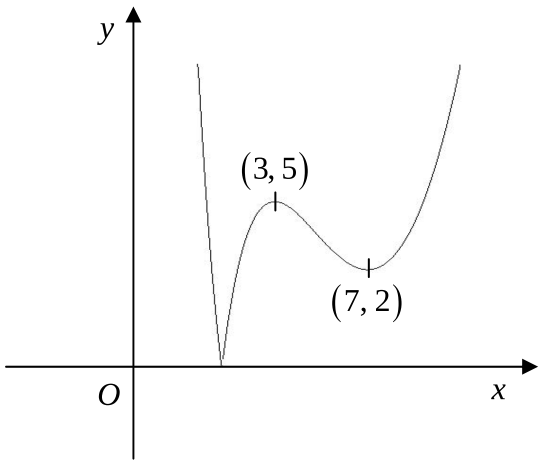
Question Number	Scheme		Marks	
4.	(a)			
				
		Shape	B1	
		$(5, 4)$	B1	
		$(-5, 4)$	B1	(3)
	(b) For the purpose of marking this paper, the graph is identical to (a)			
		Shape	B1	
		$(5, 4)$	B1	
		$(-5, 4)$	B1	(3)
	(c)			
				
		General shape – unchanged	B1	
		Translation to left	B1	
		$(4, 8)$	B1	
		$(-6, -8)$	B1	(4)
	In all parts of this question ignore any drawing outside the domains shown in the diagrams above.			**[10]**

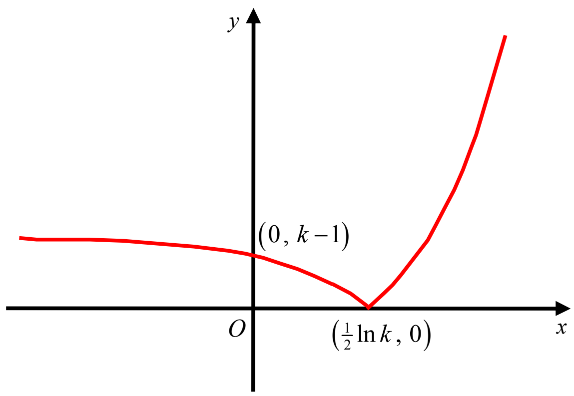
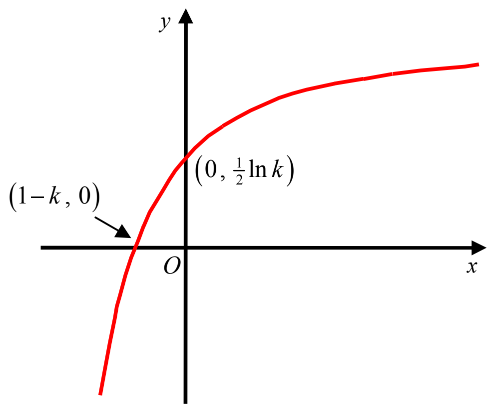
Question Number	Scheme	Marks
8.	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3 - 4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3 - 1}{1-2x^3}$ * cso	A1 (4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3 - 1 = 0$ Attempting solution of numerator = 0	M1
	$x = \frac{1}{2}$ Correct answer and no additional answers	A1 (2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$	M1 A1
	$= \frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find *y*.	M1
	$x = 0,\ y = -1$	A1 (5) **[13]**

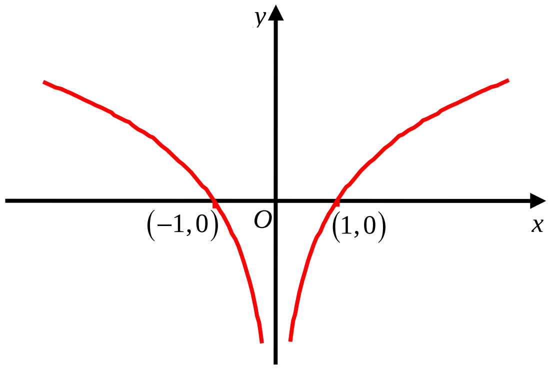
Question Number	Scheme	Marks
3.	(a) 	
	 shape	B1
	Vertices correctly placed	B1 (2)
	(b) 	
	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
	(c) $P:(-1, 2)$	B1
	$Q:(0, 1)$	B1
	$R:(1, 0)$	B1 (3)
	(d) $x > -1;\quad 2 - x - 1 = \frac{1}{2}x$	M1 A1
	Leading to $x = \frac{2}{3}$	A1
	$x < -1;\quad 2 + x + 1 = \frac{1}{2}x$	M1
	Leading to $x = -6$	A1 (5)
		[12]

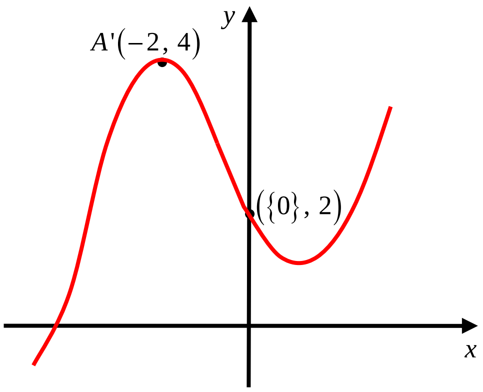
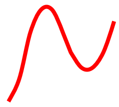
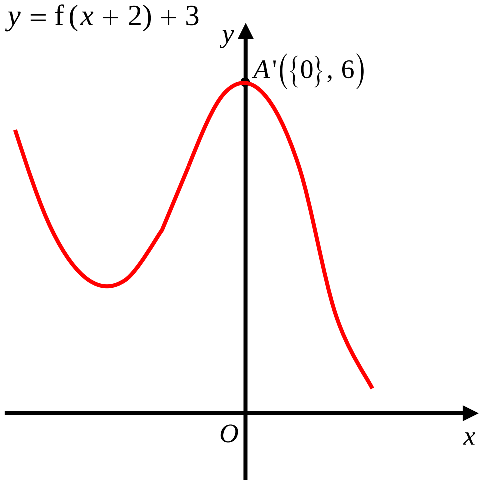
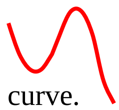
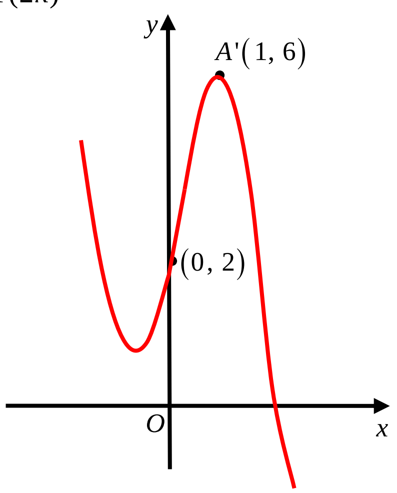
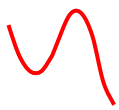
Question Number	Scheme	Marks
4.	(a) $x^2-2x-3=(x-3)(x+1)$	B1
	$\mathrm{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ * cso	A1 (4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<\mathrm{f}(x)<\dfrac{1}{4}$ etc.	B1 B1 (2)
	(c) Let $y=\mathrm{f}(x)$ $y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$\mathrm{f}^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$ ft their part (b)	B1 ft (3)
	(d) $\mathrm{fg}(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		[12]

Question Number	Scheme		Marks	
3.	(a)			
		Shape	B1	
		$(3, 6)$	B1	
		$(7, 0)$	B1	**(3)**
	(b)			
		Shape	B1	
		$(3, 5)$	B1	
		$(7, 2)$	B1	**(3)**
				[6]

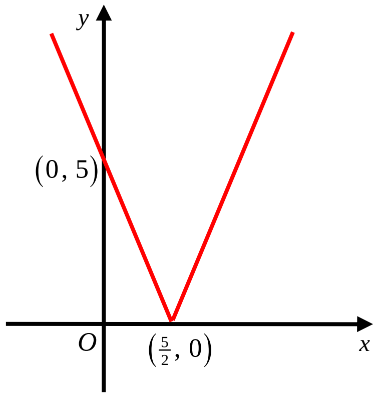
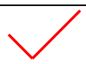
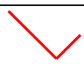
Question Number	Scheme	Marks
5.	(a) $\mathrm{g}(x) \geq 1$	B1 **(1)**
	(b) $\mathrm{f\,g}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\mathrm{e}^{x^2}$ ∗	A1 **(2)**
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}\right)$	
	(c) $\mathrm{fg}(x) \geq 3$	B1 **(1)**
	(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\mathrm{e}^{x^2}\right) = 2x + 6x\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\mathrm{e}^{x^2} = x^2\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0$, $\quad 6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 **(6)**
		[10]

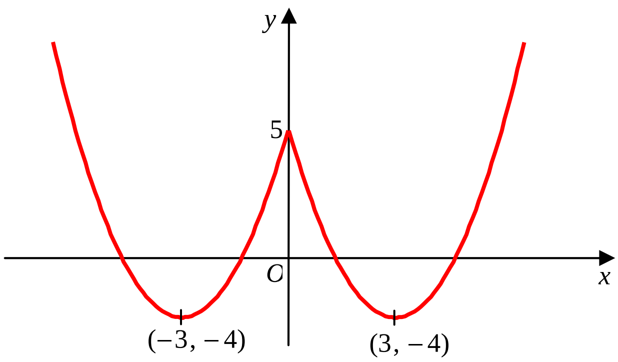
Question Number	Scheme		Marks
Q5 (a)		Curve retains shape when $x > \frac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$	B1
		$(0,\, k-1)$ and $\left(\frac{1}{2}\ln k,\, 0\right)$ marked in the correct positions.	B1
			(3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\, 0)$ and $\left(0,\, \frac{1}{2}\ln k\right)$	B1
			(2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$.	B1
			(1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	$\underline{\text{A1}}$ cao
			(3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 $\sqrt{}$
			(1)
			[10]

Question Number	Scheme		Marks
Q5	$y = \ln\lvert x\rvert$		
		Right-hand branch in quadrants 4 and 1. Correct shape.	B1
		Left-hand branch in quadrants 2 and 3. Correct shape.	B1
		Completely correct sketch and both $(-1,\{0\})$ and $(1,\{0\})$	B1
			(3)
			[3]

Question Number	Scheme	Marks
Q6 (i)	$y = \mathrm{f}(-x) + 1$	
		
	Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\}, 2)$ or $A'(-2, 4)$	B1
	Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x + 2) + 3$	
		
	Any translation  of the original curve.	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)
(iii)	$y = 2\mathrm{f}(2x)$	
		
	Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
	Either $(\{0\}, 2)$ or $A'(1, 6)$	B1
	Both $(\{0\}, 2)$ and $A'(1, 6)$	B1
		(3)
		[9]

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}\ \{=51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3+7x+2\ =\ln 15$	$x\ln 3+7x+2\ =\ln 15$	A1 oe
	$x(\ln 3+7)\ \ =-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}\ \{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x)=e^{2x}+3,\ x\in\mathbb{R}$		
	$y=e^{2x}+3 \Rightarrow y-3=e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y-3)=2x$ $\Rightarrow \frac{1}{2}\ln(y-3)=x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x)=\underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y)=\frac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> **cao**
	$f^{-1}(x)$: Domain: $\underline{x>3}$ or $\underline{(3,\infty)}$	Either $\underline{x>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Domain}>3}$.	B1
			(4)
(b)	$g(x)=\ln(x-1),\ x\in\mathbb{R},\ x>1$		
	$fg(x)=e^{2\ln(x-1)}+3\ \ \{=(x-1)^2+3\}$	An attempt to put function g into function f.	M1
		$e^{2\ln(x-1)}+3$ or $(x-1)^2+3$ or x^2-2x+4.	A1 isw
	$fg(x)$: Range: $\underline{y>3}$ or $\underline{(3,\infty)}$	Either $\underline{y>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Range}>3}$ or $\underline{fg(x)>3}$.	B1
			(3)
			[15]

Question Number	Scheme	Marks
4. (a)		M1A1
		(2)
(b)	$\underline{x = 20}$	B1
	$2x - 5 = -(15 + x)$; $\Rightarrow \underline{x = -\frac{10}{3}}$	M1;A1 oe.
		(3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3) - 5 \rvert ; = \lvert -11 \rvert = 11$	M1;A1
		(2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\text{min}} = -3$	M1
	Either $\text{g}_{\text{min}} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$	B1
	$\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	A1
		(3)
		[10]
	(a) M1: V or  or  graph with vertex on the x-axis. A1: $\left(\frac{5}{2}, \{0\}\right)$ and $\left(\{0\}, 5\right)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$ (c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \lvert 2x - 5 \rvert$ or for inserting $x = 2$ into $\lvert 2(x^2 - 4x + 1) - 5 \rvert$. There must be evidence of the modulus being applied. (d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\text{min}} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $\text{f}(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$. A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0. **Note that:** $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that**: $-3 \geqslant \text{g}(x) \geqslant 6$ is A0. **Note that:** $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0. **Note that for the final Accuracy Mark**: If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0. If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.	

Question Number	Scheme	Marks
6. (a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1
		(4)
(b)		B1 B1 B1
		(3)
(c)	$\text{f}(x) = (x - 3)^2 - 4$ or $\text{f}(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1)
		[10]
	(b) B1: Correct shape for $x \geqslant 0$, with the curve meeting the positive *y*-axis and the turning point is found below the *x*-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the *y*-axis or correct shape of curve for $x < 0$. **Note:** The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive *y*-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the *y*-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the *y*-axis. (c) M1: Either states $\text{f}(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $\text{f}(x) = x^2 + ax + b$, with $\text{f}(0) = 5$ and $\text{f}(3) = -4$ to find both *a* and *b*. A1: Either $(x - 3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $\text{f}(0) = 5$ and also $\text{f}(6) = 5$. Or: One *y*-coordinate has 2 corresponding *x*-coordinates {and therefore cannot have an inverse}.	