Leave blank

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

$$\begin{array}{r|l} & 2x^2 - 1 \\ \hline x^2 - 1 & 2x^4 + 0x^3 - 3x^2 + x + 1 \\ & 2x^4 \qquad\quad - 2x^2 \\ \hline & \qquad\quad -x^2 + x + 1 \\ & \qquad\quad -x^2 \qquad + 1 \\ \hline & \qquad\qquad +x \end{array}$$

$$\equiv 2x^2 + 0x - 1 + \frac{x + 0}{x^2 - 1}$$

$a = 2$, $b = 0$, $c = -1$, $d = 1$, $e = 0$

The new syllabus requires division by linear factors only. However, dividing by a quadratic factor is no more difficult.

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4. The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}, \quad x > 3.$ **(4)**

(b) Find the range of f. **(2)**

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(c) Find $f^{-1}(x)$. State the domain of this inverse function. **(3)**

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}$. **(3)**

a) $f(x) = \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}$

$= \frac{2(x-1)}{(x+1)(x-3)} - \frac{1}{x-3}$

$= \frac{2(x-1) - 1(x+1)}{(x+1)(x-3)}$

$= \frac{2x-2-x-1}{(x+1)(x-3)}$

$= \frac{(x-3)}{(x+1)(x-3)}$

$f(x) = \frac{1}{x+1}$

b) Range of $f(x)$ for $x>3$

$0 < f(x) < \frac{1}{4}$

c) Let $y = \frac{1}{1+x}$

Swap variables $x = \frac{1}{1+y}$

$(1+y)x = 1$

$1+y = \frac{1}{x}$

$y = \frac{1}{x} - 1$ $\quad f^{-1}(x) = \frac{1}{x} - 1$

Domain $0 < x < \frac{1}{4}$

d) $g(x) = 2x^2-3$

$fg(x) = f(2x^2-3)$

$= \frac{1}{2x^2-3+1}$

Solve

$\frac{1}{2x^2-2} = \frac{1}{8}$

$8 = 2x^2 - 2$

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4d cont

$$4 = x^2 - 1$$

$$5 = x^2$$

$$\pm\sqrt{5} = x$$

$$x = \pm\sqrt{5}$$

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2.
$$\mathrm{f}(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express $\mathrm{f}(x)$ as a single fraction in its simplest form. **(4)**

(b) Hence show that $\mathrm{f}'(x) = \dfrac{2}{(x-3)^2}$ **(3)**

a)
$$f(x) = \frac{2\cancel{(x+1)}}{\cancel{(x+1)}(x-3)} - \frac{x+1}{x-3}$$

$$= \frac{2-(x+1)}{x-3}$$

$$= \frac{1-x}{x-3}$$

b)
$$f'(x) = \frac{(x-3)(-1)-(1-x)(1)}{(x-3)^2}$$

$$= \frac{-x+3-1+x}{(x-3)^2}$$

$$= \frac{2}{(x-3)^2}$$

H 3 1 1 2 3 A 0 4 2 8

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7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of x for which $g'(x) = 1$ **(4)**

a) $$f(x) = \frac{(x-2)(x+4) - 2(x-2) + (x-8)}{(x-2)(x+4)}$$

$$= \frac{x^2 - 2x + 4x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$$

$$= \frac{x^2 + x - 12}{(x-2)(x+4)}$$

$$= \frac{\cancel{(x+4)}(x-3)}{(x-2)\cancel{(x+4)}}$$

$$= \frac{x-3}{x-2}$$

b) $$g(x) = \frac{e^x - 3}{e^x - 2}$$

$$g'(x) = \frac{(e^x - 2)(e^x) - (e^x - 3)e^x}{(e^x - 2)^2}$$

7b cont

$$g'(x) = \frac{e^x(e^x - 2 - e^x + 3)}{(e^x-2)^2}$$

$$= \frac{e^x(1)}{(e^x-2)^2}$$

$$= \frac{e^x}{(e^x-2)^2}$$

7c) $g'(x) = 1 \Rightarrow e^x = (e^x-2)^2$

$$e^x = e^{2x} - 4e^x + 4$$

$$0 = e^{2x} - 5e^x + 4$$

$$0 = (e^x - 4)(e^x - 1)$$

Either $e^x - 4 = 0$ or $e^x - 1 = 0$

$e^x = 4$ $\qquad$ $e^x = 1$

$x = \ln 4$ $\qquad$ $x = 0$

Leave blank

1. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

$$= \frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$$

$$= \frac{(x+1)(3x+1) - 3(x^2-1)}{3(x^2-1)(3x+1)}$$

$$= \frac{3x^2+3x+x+1-3x^2+3}{3(x^2-1)(3x+1)}$$

$$= \frac{4x+4}{3(x^2-1)(3x+1)}$$

$$= \frac{4\cancel{(x+1)}}{3\cancel{(x+1)}(x-1)(3x+1)}$$

$$= \frac{4}{3(x-1)(3x+1)}$$

N 3 5 3 8 1 A 0 2 2 8

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8. (a) Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}$$

(3)

Given that

$$\ln(2x^2+9x-5)=1+\ln(x^2+2x-15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

a)
$$\frac{(2x-1)\cancel{(x+5)}}{(x-3)\cancel{(x+5)}}$$

$$= \frac{2x-1}{x-3}$$

b)
$$\ln(2x^2+9x-5)-\ln(x^2+2x-15)=1$$

$$\ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right)=1$$

$$\ln\left(\frac{2x-1}{x-3}\right)=1$$

$$\frac{2x-1}{x-3}=e$$

$$2x-1=e(x-3)$$
$$2x-1=ex-3e$$
$$3e-1=ex-2x$$
$$3e-1=x(e-2)$$

$$x=\frac{3e-1}{e-2}$$