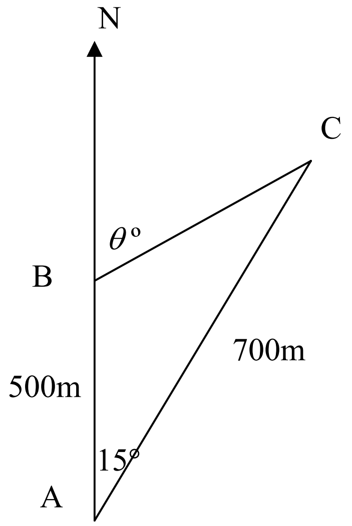
6.



6.	$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15°$ (= 63851.92…)	M1 A1
	BC = 253 awrt	A1 (3)
(a)	$\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$	M1
	$\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B (134.2°) dep	M1
(b)	θ = 180° − candidate's angle B (Dep. on first M only, B can be acute)	M1
	$\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) [46 needs to be from correct working]	A1 (4) **[7]**

Notes:

(a) If use cos 15° = ..., then A1 not scored until written as BC^2 = … correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC

Finding value for BX and CX and using Pythagoras M1

$BC^2 = (500\sin 15^\circ)^2 + (700 - 500\cos 15^\circ)^2$ A1

BC = 253 awrt A1

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M))

(i) $\cos B = \frac{500^2 + \text{candidate's} BC^2 - 700^2}{2\text{x}500\text{xcandidate's} BC}$ or $700^2 = 500^2 + BC_c^{\ 2} - 2x500xBC_c$ M1

Finding angle B M1, then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)

$\tan CBX = \frac{700 - valueforAX}{valueforBX}$ M1

Finding value for $\angle CBX$ ($\approx 59°$) M1

$\theta = [180^\circ - (75^\circ + candidate's\ \angle CBX)]$ M1

(iii) Using sine rule (or cos rule) to find C first:

Correct use of sine or cos rule for C M1, Finding value for C M1

Either B =180° – (15° + candidate's C) or θ = (15° + candidate's C) M1

(iv) $700\cos 15^\circ = 500 + BC\cos\theta$ M2 {first two Ms earned in this case}

Solving for θ; θ = 45.8 (allow 46 or5.7, 45.8, 45.9 M1;A1

June 2008
Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
7.	(a) $r\theta = 7\times 0.8 = 5.6$ (cm)	M1 A1 (2)
	(b) $\frac{1}{2}r^2\theta = \frac{1}{2}\times 7^2\times 0.8 = 19.6$ (cm^2)	M1 A1 (2)
	(c) $BD^2 = 7^2 + (\text{their } AD)^2 - (2\times 7\times(\text{their } AD)\times\cos 0.8)$	M1
	$BD^2 = 7^2 + 3.5^2 - (2\times 7\times 3.5\times\cos 0.8)$ (or awrt 46° for the angle)	A1
	$(BD = 5.21)$ Perimeter = (their DC) + "5.6" + "5.21" = 14.3 (cm) (Accept awrt)	M1 A1 (4)
	(d) $\Delta ABD = \frac{1}{2}\times 7\times(\text{their } AD)\times\sin 0.8$ (or awrt 46° for the angle) (ft their AD)	M1 A1ft
	$(= 8.78\ldots)$	
	(If the correct formula $\frac{1}{2}ab\sin C$ is quoted the use of any two of the sides of ΔABD as a and b scores the M mark).	
	Area = "19.6" – "8.78…" = 10.8 (cm^2) (Accept awrt)	M1 A1 (4)
		12
	Units (cm or cm^2) are not required in any of the answers. (a) and (b): Correct answers without working score both marks. (a) M: Use of $r\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (b) M: Use of $\frac{1}{2}r^2\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (c) 1st M: Use of the (correct) cosine rule formula to find BD^2 or BD. Any other methods need to be complete methods to find BD^2 or BD. 2nd M: Adding their DC to their arc BC and their BD. <u>Beware</u>: If 0.8 is used, but calculator is in degree mode, this can still earn M1 A1 (for the required expression), but this gives $BD = 3.50\ldots$ so the perimeter may appear as 3.5 + 5.6 + 3.5 (earning M1 A0). (d) 1st M: Use of the (correct) area formula to find ΔABD. Any other methods need to be complete methods to find ΔABD. 2nd M: Subtracting their ΔABD from their sector ABC. Using segment formula $\frac{1}{2}r^2(\theta - \sin\theta)$ scores no marks in part (d).	

Question Number	Scheme	Marks
7 (a)	$\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2\times 2.2 = 39.6 \quad (\text{cm}^2)$	M1 A1 (2)
(b)	$\left(\frac{2\pi - 2.2}{2} =\right)\ \pi - 1.1 = 2.04 \quad (\text{rad})$	M1 A1 (2)
	(c) $\Delta DAC = \frac{1}{2}\times 6\times 4\sin 2.04 \qquad (\approx 10.7)$	M1 A1ft
	Total area = sector + 2 triangles = 61 $\quad (\text{cm}^2)$	M1 A1 (4) **[8]**
(a)	**M1:** Needs θ in radians for this formula. Could convert to degrees and use degrees formula. **A1:** Does not need units. Answer should be 39.6 exactly. Answer with no working is **M1 A1**. This **M1A1** can only be awarded in part (a).	
(b)	**M1:** Needs full method to give angle in radians **A1:** Allow answers which round to 2.04 (Just writes 2.04 – no working is 2/2)	
(c)	**M1:** Use $\frac{1}{2}\times 6\times 4\sin A$ (if any other triangle formula e.g. $\frac{1}{2}b\times h$ is used the method must be complete for this mark) (No value needed for A, but should not be using 2.2) **A1:** ft the value obtained in part (b) – need not be evaluated- could be in degrees **M1:** Uses Total area = sector + 2 triangles or other complete method **A1:** Allow answers which round to 61. (Do not need units) Special case degrees: Could get M0A0, M0A0, M1A1M1A0 Special case: Use $\Delta\, BDC - \Delta\, BAC$ Both areas needed for first **M1** Total area = sector + area found is second **M1** **NB** Just finding lengths BD, DC, and angle BDC then assuming area BDC is a sector to find area BDC is 0/4	

Question Number	Scheme	Marks
Q9 (a)	(Arc length =) $r\theta = r \times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).	B1
	(Sector area =) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).	B1
	Surface area = 2 sectors + 2 rectangles + curved face (= $r^2 + 3rh$) (See notes below for what is allowed here)	M1
	Volume = $300 = \frac{1}{2}r^2h$	B1
	Sub for h: $S = r^2 + 3\times\frac{600}{r} = r^2 + \frac{1800}{r}$ (*)	A1cso (5)
(b)	$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$	M1A1
	$\frac{dS}{dr} = 0 \Rightarrow r^3 = ...,\quad r = \sqrt[3]{900}$, or AWRT 9.7 (NOT -9.7 or ± 9.7)	M1, A1 (4)
(c)	$\frac{d^2S}{dr^2} = ...$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum	M1, A1ft (2)
(d)	$S_{min} = (9.65...)^2 + \frac{1800}{9.65...}$	
	(Using their value of r, however found, in the <u>given</u> S formula)	M1
	$= 279.65\ldots$ (AWRT: 280) (Dependent on full marks in part (b))	A1 (2) **[13]**
(a)	M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\theta$) term and an rh (or $rh\theta$) term.	
(b)	<u>In parts (b), (c) and (d), ignore labelling of parts</u> 1st M1 for attempt at differentiation (one term is sufficient) $r^n \to kr^{n-1}$ 2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3 = ...$ (depending upon their 'changed function', this could be $r = ...$ or $r^2 = ...$, etc., but the algebra <u>must deal with a negative power</u> of r and should be sound apart from possible <u>sign</u> errors, so that $r^n = ...$ is consistent with their derivative).	
(c)	M1 for attempting second derivative (one term is sufficient) $r^n \to kr^{n-1}$, <u>and considering its sign</u>. Substitution of a value of r is not required. (<u>Equating it to zero is M0</u>). A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum. <u>Alternative</u>: M1: Find <u>value</u> of $\frac{dS}{dr}$ on each side of their value of r and consider sign. A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum. <u>Alternative</u>: M1: Find <u>value</u> of S on each side of their value of r and compare with their 279.65. A1ft: Indicate that both values are more than 279.65, and conclude minimum.	

Question Number	Scheme	Marks
Q4 (a)	**Either** $\frac{\sin(A\hat{C}B)}{5} = \frac{\sin 0.6}{4}$ $\therefore\ A\hat{C}B = \arcsin(0.7058...)$ $= [0.7835..\ \text{ or }\ 2.358]$ Use angles of triangle $A\hat{B}C = \pi - 0.6 - A\hat{C}B$ (But as AC is the longest side so) $A\hat{B}C = 1.76$ **(*)**(3sf) [Allow $100.7^\circ \rightarrow 1.76$] In degrees $0.6 = 34.377^\circ$, $\text{A}\hat{\text{C}}\text{B} = 44.9^\circ$ **or** $4^2 = b^2 + 5^2 - 2 \times b \times 5\cos 0.6$ $\therefore b = \frac{10\cos 0.6 \pm \sqrt{(100\cos^2 0.6 - 36)}}{2}$ $= [6.96\ \text{ or }\ 1.29]$ Use sine / cosine rule with value for b $\sin B = \frac{\sin 0.6}{4} \times b$ or $\cos B = \frac{25 + 16 - b^2}{40}$ (But as AC is the longest side so) $A\hat{B}C = 1.76$ **(*)**(3sf)	M1 M1 M1, A1 (4)
(b)	$\left\lvert C\hat{B}D = \pi - 1.76 = 1.38 \right\rvert$ Sector area = $\frac{1}{2} \times 4^2 \times (\pi - 1.76) = [11.0 \sim 11.1]$ $\frac{1}{2} \times 4^2 \times 79.3$ is M0 Area of $\Delta ABC = \frac{1}{2} \times 5 \times 4 \times \sin(1.76) = [9.8]$ or $\frac{1}{2} \times 5 \times 4 \times \sin 101$ Required area = awrt 20.8 or 20.9 or 21.0 or gives 21 (2sf) after correct work.	M1 M1 A1 (3) **[7]**
(a) (b)	1st M1 for correct use of sine rule to find *ACB* or cosine rule to find *b* (M0 for ABC here or for use of sin x where *x* could be *ABC*) 2nd M1 for a correct expression for angle *ACB* (This mark may be implied by .7835 or by arcsin (.7058)) and needs accuracy. In second method this M1 is for correct expression for *b* – may be implied by 6.96. [Note $10\cos 0.6 \approx 8.3$] (do not need two answers) 3rd M1 for a correct method to get angle *ABC* in method (i) or sin*ABC* or cos*ABC* , in method (ii) (If sin *B* >1, can have M1A0) A1cso for correct work leading to 1.76 3sf . Do not need to see angle 0.1835 considered and rejected. 1st M1 for a correct expression for sector area or a value in the range 11.0 – 11.1 2nd M1 for a correct expression for the area of the triangle or a value of 9.8 Ignore 0.31 (working in degrees) as subsequent work. A1 for answers which round to 20.8 or 20.9 or 21.0. No need to see units.	
(a)	Special case If answer 1.76 is assumed then usual mark is M0 M0 M0 A0. A Fully checked method may be worth M1 M1 M0 A0. A maximum of 2 marks. The mark is either 2 or 0. Either **M1** for $A\hat{C}B$ is found to be 0,7816 (angles of triangle) then **M1** for checking $\frac{\sin(A\hat{C}B)}{5} = \frac{\sin 0.6}{4}$ with conclusion giving numerical answers **This gives a maximum mark of 2/4** OR **M1** for *b* is found to be 6.97 (cosine rule) **M1** for checking $\frac{\sin(ABC)}{b} = \frac{\sin 0.6}{4}$ with conclusion giving numerical answers **This gives a maximum mark of 2/4** Candidates making this assumption need a complete method. They cannot earn M1M0. So the score will be 0 or 2 for part (a). Circular arguments earn 0/4.	

Question Number	Scheme	Marks
6	(a) $r\theta = 9 \times 0.7 = 6.3$ (Also allow 6.30, or awrt 6.30)	M1 A1 (2)
	(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 81 \times 0.7 = 28.35$ (Also allow 28.3 or 28.4, or awrt 28.3 or 28.4) (Condone 28.35^2 written instead of $28.35\,\text{cm}^2$)	M1 A1 (2)
	(c) $\tan 0.7 = \frac{AC}{9}$ $AC = 7.58$ (Allow awrt) <u>NOT</u> 7.59 (see below)	M1 A1 (2)
	(d) Area of triangle $AOC = \frac{1}{2}(9 \times \text{their } AC)$ (or other complete method) Area of R = "34.11" – "28.35" (triangle – sector) or (sector – triangle) (needs a <u>value</u> for each) = 5.76 (Allow awrt)	M1 M1 A1 (3) **9**
	(a) M: Use of $r\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (b) M: Use of $\frac{1}{2}r^2\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (c) M: Other methods must be fully correct, e.g. $\frac{AC}{\sin 0.7} = \frac{9}{\sin\left(\frac{\pi}{2} - 0.7\right)}$ $(\pi - 0.7)$ instead of $\left(\frac{\pi}{2} - 0.7\right)$ here is <u>not</u> a fully correct method. <u>Premature approximation (e.g. taking angle *C* as 0.87 radians)</u>: This will often result in loss of A marks, e.g. $AC = 7.59$ in (c) is A0.	