4.	(a)	$3\sin^2\theta - 2\cos^2\theta = 1$	
		$3\sin^2\theta - 2(1 - \sin^2\theta) = 1$ (M1: Use of $\sin^2\theta + \cos^2\theta = 1$)	M1
		$3\sin^2\theta - 2 + 2\sin^2\theta = 1$	
		$5\sin^2\theta = 3$ cso **AG**	A1 (2)
	(b)	$\sin^2\theta = \frac{3}{5}$, so $\sin\theta = (\pm)\sqrt{0.6}$	M1
		Attempt to solve both $\sin\theta$ = +.. and $\sin\theta$ = – (may be implied by later work) M1	
		$\theta = 50.7685^\circ$ awrt $\theta = 50.8°$ (dependent on first M1 only)	A1
		θ (= $180^\circ - 50.7685_c{}^\circ$); = 129.23…° awrt 129.2°	M1; A1 √
		[f.t. dependent on first M and 3rd M]	
		$\sin\theta = -\sqrt{0.6}$	
		$\theta = 230.785^\circ$ and 309.23152° awrt 230.8°, 309.2° (both)	M1A1 (7)
			[9]

Notes:

(a) N.B: **AG**; need to see at least one line of working after substituting $\cos^2\theta$.

(b) First M1: Using $5\sin^2\theta = 3$ to find value for $\sin\theta$ or θ

Second M1: Considering the – value for $\sin\theta$. (usually later)

First A1: Given for awrt 50.8°. Not dependent on second M.

Third M1: For $(180 - 50.8_c)°$, need not see written down

Final M1: **Dependent** on second M (but may be implied by answers)

For (180 + candidate's 50.8)° **or** $(360 - 50.8_c)°$ **or equiv**.

Final A1: Requires both values. (no follow through)

[Finds $\cos^2\theta = k$ (k = 2/5) and so $\cos\theta = (\pm)$...M1, then mark equivalently]

June 2008
Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
9.	(a) 45 (α) (This mark can be implied by an answer 65) $180-\alpha$, Add 20 (for at least one angle) 65 155	B1 M1, M1 A1 (4)
	(b) 120 or 240 (β): (This mark can be implied by an answer 40 or 80) (Could be achieved by working with 60, using $180-60$ and/or $180+60$) $360-\beta$, $360+\beta$ (or 120 + an angle that has been divided by 3) Dividing by 3 (for at least one angle) 40 80 160 200 280 320 First A1: at least 3 correct	B1 M1, M1 M1 A1 A1 (6) **10**
	(a) Extra solution(s) in range: Loses the A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 65 (only correct solution) will score B1 M0 M1 A0 (2 marks) 65 and 115 will score B1 M0 M1 A0 (2 marks) 44.99 (or similar) for α is B0, and 64.99, 155.01 (or similar) is A0. (b) Extra solution(s) in range: Loses the final A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 40 (only correct solution) will score B1 M0 M0 M1 A0 A0 (2 marks) 40 and 80 (only correct solutions) B1 M1 M0 M1 A0 A0 (3 marks) 40 and 320 (only correct solutions) B1 M0 M0 M1 A0 A0 (2 marks) Answers without working: Full marks can be given (in both parts), B and M marks by implication. Answers given in radians: Deduct a maximum of 2 marks (misread) from B and A marks. (Deduct these at first and second occurrence.) Answers that begin with statements such as $\sin(x-20)=\sin x-\sin 20$ or $\cos x=-\frac{1}{6}$, then go on to find a value of 'α' or 'β', however badly, can continue to earn the first M mark in either part, but will score no further marks. Possible misread: $\cos 3x=\frac{1}{2}$, giving 20, 100, 140, 220, 260, 340 Could score up to 4 marks B0 M1 M1 M1 A0 A1 for the above answers.	

Question Number	Scheme	Marks
8		
(a)	$4(1-\cos^2 x)+9\cos x-6=0 \qquad 4\cos^2 x-9\cos x+2=0$ (*)	M1 A1 (2)
(b)	$(4\cos x-1)(\cos x-2)=0 \qquad \cos x=...,\ \frac{1}{4}$	M1 A1
	$x=75.5 \qquad (\alpha)$	B1
	$360-\alpha, \qquad 360+\alpha$ or $720-\alpha$	M1, M1
	284.5, 435.5, 644.5	A1 (6) **[8]**
(a)	**M1:** Uses $\sin^2 x=1-\cos^2 x$ (may omit bracket) **not** $\sin^2 x=\cos^2 x-1$ **A1:** Obtains the printed answer without error – **must have = 0**	
(b)	**M1:** Solves the quadratic with usual conventions **A1:** Obtains ¼ accurately- ignore extra answer 2 but penalise e.g. -2. **B1:** allow answers which round to 75.5 **M1:** $360-\alpha$ ft their value, **M1:** $360+\alpha$ ft their value or $720-\alpha$ ft **A1:** Three **and only three** correct exact answers in the range achieves the mark	
Special cases	In part (b) Error in solving quadratic (4cosx-1)(cosx+2) Could yield, **M1A0B1M1M1A1** losing one mark for the error Works in radians: Complete work in radians :Obtains 1.3 **B0**. Then allow **M1 M1** for $2\pi-\alpha$, $2\pi+\alpha$ or $4\pi-\alpha$ Then gets 5.0, 7.6, 11.3 **A0 so 2/4** Mixed answer 1.3, 360 – 1.3, 360 + 1.3, 720 – 1.3 still gets **B0M1M1A0**	

Question Number	Scheme	Marks
Q7 (i)	$\tan\theta = -1 \Rightarrow \quad \theta = -45, \quad 135$	B1, B1ft
	$\sin\theta = \frac{2}{5} \Rightarrow \quad \theta = 23.6, \quad 156.4$ (AWRT: 24, 156)	B1, B1ft (4)
(ii)	$4\sin x = \dfrac{3\sin x}{\cos x}$	M1
	$4\sin x\cos x = 3\sin x \quad \Rightarrow \quad \sin x(4\cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x(16\sin^2 x - 7) = 0$, $(16\cos^2 x - 9)(\cos^2 x - 1) = 0$	M1
	$x = 0,\ 180$ <u>seen</u>	B1, B1
	$x = 41.4,\ 318.6$ (AWRT: 41, 319)	B1, B1ft (6)
		[10]

(i)

1st B1 for -45 seen (α, where $|\alpha| < 90$)

2nd B1 for 135 seen, <u>or ft</u> $(180 + \alpha)$ if α is negative, or $(\alpha - 180)$ if α is positive.
If $\tan\theta = k$ is obtained from <u>wrong working</u>, 2nd B1ft is still available.

3rd B1 for awrt 24 (β, where $|\beta| < 90$)

4th B1 for awrt 156, <u>or ft</u> $(180 - \beta)$ if β is positive, or $-(180 + \beta)$ if β is negative.
If $\sin\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.

(ii)

1st M1 for use of $\tan x = \dfrac{\sin x}{\cos x}$. Condone $\dfrac{3\sin x}{3\cos x}$.

2nd M1 for correct work leading to 2 factors (may be implied).

1st B1 for 0, 2nd B1 for 180.

3rd B1 for awrt 41 (γ, where $|\gamma| < 180$)

4th B1 for awrt 319, <u>or ft</u> $(360 - \gamma)$.
If $\cos\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.

N.B. Losing $\sin x = 0$ usually gives a maximum of 3 marks M1M0B0B0B1B1

<u>Alternative:</u> (squaring both sides)

1st M1 for squaring both sides and using a 'quadratic' identity.
e.g. $16\sin^2\theta = 9(\sec^2\theta - 1)$

2nd M1 for reaching a factorised form.
e.g. $(16\cos^2\theta - 9)(\cos^2\theta - 1) = 0$

Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme.

<u>For both parts of the question</u>:

<u>Extra solutions outside required range</u>: Ignore

<u>Extra solutions inside required range</u>:
For each <u>pair</u> of B marks, the 2nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan\theta = -1 \Rightarrow \theta = 45, -45, 135$ is B1 B0

<u>Answers in radians</u>:
Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).

Question Number	Scheme	Marks
Q2 (a)	$5\sin x = 1 + 2\left(1 - \sin^2 x\right)$	M1
	$2\sin^2 x + 5\sin x - 3 = 0$ **(*)**	A1cso (2)
(b)	$(2s-1)(s+3) = 0$ **giving** s =	M1
	$[\sin x = -3$ has no solution$]$ **so** $\sin x = \frac{1}{2}$	A1
	$\therefore \quad x = 30,\ 150$	B1, B1ft (4) **[6]**

(a)

M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$)
A1 need 3 term quadratic printed in any order with =0 included

(b)

M1 for attempt to solve given quadratic (usual rules for solving quadratics)
(can use any variable here, *s*, *y*, *x*, or sin*x*)
A1 requires no incorrect work seen and is for $\sin x = \frac{1}{2}$ **or** $x = \sin^{-1}\frac{1}{2}$
$y = \frac{1}{2}$ is A0 (unless followed by $x = 30$)
B1 for 30 (α) not dependent on method
2nd B1 for 180 - α provided in required range (otherwise 540 - α)
Extra solutions outside required range: Ignore
Extra solutions inside required range: Lose final B1
Answers in radians: Lose final B1
S.C. Merely writes down two correct answers is M0A0B1B1
Or $\sin x = \frac{1}{2} \quad \therefore \quad x = 30,\ 150$ **is M1A1B1B1**
Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1

NB Common error is to factorise wrongly giving $(2\sin x + 1)(\sin x - 3) = 0$
$[\sin x = 3$ gives no solution$]$ $\sin x = -\frac{1}{2} \quad \Rightarrow \quad x = 210,\ 330$
This earns M1 A0 B0 B1ft

Another common error is to factorise correctly $(2\sin x - 1)(\sin x + 3) = 0$ and follow this with $\sin x = \frac{1}{2}$, $\sin x = 3$ then $x = 30^\circ, 150^\circ$
This would be M1 A0 B1 B1

Question Number	Scheme	Marks
5	(a) $\tan\theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found) Requires the correct value with no incorrect working seen.	B1 (1)
	(b) awrt 21.8 (α) (Also allow awrt 68.2, ft from $\tan\theta = \frac{5}{2}$ in (a), but no other ft) (This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)	B1
	$180 + \alpha$ $(= 201.8)$, or $90 + (\alpha/2)$ (if division by 2 has already occurred) (α found from $\tan 2x = ...$ or $\tan x = ...$ or $\sin 2x = \pm...$ or $\cos 2x = \pm...$)	M1
	$360 + \alpha$ $(= 381.8)$, or $180 + (\alpha/2)$ (α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$) OR $540 + \alpha$ $(= 561.8)$, or $270 + (\alpha/2)$ (α found from $\tan 2x = ...$)	M1
	Dividing at least one of the angles by 2 (α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$)	M1
	$x = 10.9,\ 100.9,\ 190.9,\ 280.9$ (Allow awrt)	A1 (5) **6**

(b) Extra solution(s) in range: Loses the final A mark.
Extra solutions outside range: Ignore (whether correct or not).
Common answers:
10.9 and 100.9 would score B1 M1 M0 M1 A0 (Ensure that these M marks are awarded)
10.9 and 190.9 would score B1 M0 M1 M1 A0 (Ensure that these M marks are awarded)
Alternatives:

(i) $2\cos 2x - 5\sin 2x = 0$ $R\cos(2x + \lambda) = 0$ $\lambda = 68.2 \Rightarrow 2x + 68.2 = 90$ B1
$2x + \lambda = 270$ M1
$2x + \lambda = 450$ or $2x + \lambda = 630$ M1
Subtracting λ and dividing by 2 (at least once) M1

(ii) $25\sin^2 2x = 4\cos^2 2x = 4(1 - \sin^2 2x)$
$29\sin^2 2x = 4$ $2x = 21.8$ B1
The M marks are scored as in the main scheme, but extra solutions will be likely, losing the A mark.
Using radians:
B1: Can be given for awrt 0.38 (β)
M1: For $\pi + \beta$ or $180 + \beta$
M1: For $2\pi + \beta$ or $3\pi + \beta$ (Must now be consistently radians)
M1: For dividing at least one of the angles by 2
A1: For this mark, the answers must be in degrees.
(Correct) answers only (or by graphical methods):
B and M marks can be awarded by implication, e.g.
10.9 scores B1 M0 M0 M1 A0
10.9, 100.9 scores B1 M1 M0 M1 A0
10.9, 100.9, 190.9, 280.9 scores full marks.
Using 11, etc. instead of 10.9 can still score the M marks by implication.