

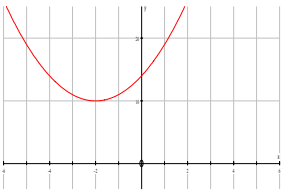
## ROOTS OF QUADRATICS

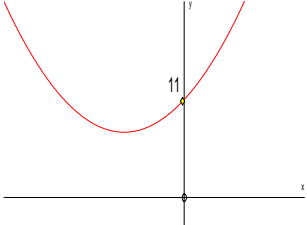
Question number	Scheme	Marks
8.	<p>(a) <math>x^2 + kx + (8 - k) = 0</math>      <math>8 - k</math> need not be bracketed</p> <p><math>b^2 - 4ac = k^2 - 4(8 - k)</math></p> <p><math>b^2 - 4ac &lt; 0 \Rightarrow k^2 + 4k - 32 &lt; 0</math>      (*)</p> <p>(b) <math>(k + 8)(k - 4) = 0</math>      <math>k = \dots</math></p> <p style="padding-left: 100px;"><math>k = -8</math>      <math>k = 4</math></p> <p>Choosing 'inside' region (between the two <math>k</math> values)</p> <p style="padding-left: 100px;"><math>-8 &lt; k &lt; 4</math>    or    <math>4 &gt; k &gt; -8</math></p>	<p>M1</p> <p>M1</p> <p>A1cso      (3)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1      (4)</p> <p style="text-align: right;"><b>7</b></p>
	<p>(a) 1<sup>st</sup> M: Using the <math>k</math> from the right hand side to form 3-term quadratic in <math>x</math> ('= 0' can be implied), or...</p> <p>attempting to complete the square <math>\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 - k (= 0)</math> or equiv., using the <math>k</math> from the right hand side.</p> <p>For either approach, <u>condone sign errors</u>.</p> <p>1<sup>st</sup> M may be implied when candidate moves straight to the discriminant.</p> <p>2<sup>nd</sup> M: Dependent on the 1<sup>st</sup> M.</p> <p>Forming expressions in <math>k</math> (with no <math>x</math>'s) by using <math>b^2</math> and <math>4ac</math>. (Usually seen as the discriminant <math>b^2 - 4ac</math>, but separate expressions are fine, and also allow the use of <math>b^2 + 4ac</math>.</p> <p>(For 'completing the square' approach, the expression must be clearly separated from the equation in <math>x</math>).</p> <p>If <math>b^2</math> and <math>4ac</math> are used in the <u>quadratic formula</u>, they must be clearly separated from the formula to score this mark.</p> <p>For any approach, <u>condone sign errors</u>.</p> <p>If the wrong statement <math>\sqrt{b^2 - 4ac} &lt; 0</math> is seen, maximum score is M1 M1 A0.</p> <p>(b) Condone the use of <math>x</math> (instead of <math>k</math>) in part (b).</p> <p>1<sup>st</sup> M: Attempt to solve a 3-term quadratic equation in <math>k</math>.</p> <p>It <u>might</u> be different from the given quadratic in part (a).</p> <p>Ignore the use of <math>&lt;</math> in solving the equation. The 1<sup>st</sup> M1 A1 can be scored if <math>-8</math> and <math>4</math> are achieved, even if stated as <math>k &lt; -8</math>, <math>k &lt; 4</math>.</p> <p><u>Allow</u> the first M1 A1 to be scored in part (a).</p> <p>N.B. '<math>k &gt; -8</math>, <math>k &lt; 4</math>' scores 2<sup>nd</sup> M1 A0</p> <p style="padding-left: 20px;"><math>'k &gt; -8</math> or <math>k &lt; 4</math>' scores 2<sup>nd</sup> M1 A0</p> <p style="padding-left: 20px;"><math>'k &gt; -8</math> and <math>k &lt; 4</math>' scores 2<sup>nd</sup> M1 A1</p> <p style="padding-left: 20px;"><math>'k = -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3</math>' scores 2<sup>nd</sup> M0 A0</p> <p>Use of <math>\leq</math> (in the answer) loses the final mark.</p>	

Question number	Scheme	Marks
8. (a)	<p>[No real roots implies <math>b^2 - 4ac &lt; 0</math> .] <math>b^2 - 4ac = q^2 - 4 \times 2q \times (-1)</math>            So <math>q^2 - 4 \times 2q \times (-1) &lt; 0</math> i.e. <math>q^2 + 8q &lt; 0</math> (*)</p>	<p>M1            A1cso (2)</p>
(b)	<p><math>q(q + 8) = 0</math> or <math>(q \pm 4)^2 \pm 16 = 0</math>  <math>(q) = 0</math> or <math>-8</math> (2 cvs)  <math>-8 &lt; q &lt; 0</math> <u>or</u> <math>q \in (-8, 0)</math> <u>or</u> <math>q &lt; 0</math> and <math>q &gt; -8</math></p>	<p>M1            A1            A1ft (3)  <b>5</b></p>
(a)	<p>M1 for attempting <math>b^2 - 4ac</math> with one of <math>b</math> or <math>a</math> correct. <math>&lt; 0</math> not needed for M1            This may be inside a square root.            A1cso for simplifying to printed result with no incorrect working or statements seen.            Need an intermediate step            e.g. <math>q^2 - 8q &lt; 0</math> or <math>q^2 - 4 \times 2q \times -1 &lt; 0</math> or <math>q^2 - 4(2q)(-1) &lt; 0</math> or <math>q^2 - 8q(-1) &lt; 0</math> or <math>q^2 - 8q \times -1 &lt; 0</math>            i.e. must have <math>\times</math> or brackets on the <math>4ac</math> term  <math>&lt; 0</math> must be seen at least one line before the final answer.</p>	
(b)	<p>M1 for factorizing or completing the square or attempting to solve <math>q^2 \pm 8q = 0</math>. A method that would lead to 2 values for <math>q</math>. The “= 0” may be implied by values appearing later.            1<sup>st</sup> A1 for <math>q = 0</math> and <math>q = -8</math>            2<sup>nd</sup> A1 for <math>-8 &lt; q &lt; 0</math>. Can follow through their cvs but must choose “inside” region.  <math>q &lt; 0, q &gt; -8</math> is A0, <math>q &lt; 0</math> or <math>q &gt; -8</math> is A0, <math>(-8, 0)</math> on its own is A0            BUT “<math>q &lt; 0</math> and <math>q &gt; -8</math>” is A1             Do not accept a number line for final mark</p>	

Question Number	Scheme	Marks
7	<p>(a) <math>b^2 - 4ac &gt; 0 \Rightarrow 16 - 4k(5 - k) &gt; 0</math> or equiv., e.g. <math>16 &gt; 4k(5 - k)</math></p> <p>So <math>k^2 - 5k + 4 &gt; 0</math> (Allow any order of terms, e.g. <math>4 - 5k + k^2 &gt; 0</math>) (*)</p> <p>(b) <u>Critical Values</u> <math>(k - 4)(k - 1) = 0</math> <math>k = \dots</math></p> <p style="text-align: center;"><math>k = 1</math> or <math>4</math></p> <p style="text-align: right;">Choosing "outside" region</p> <p style="text-align: center;"><u><math>k &lt; 1</math> or <math>k &gt; 4</math></u></p>	<p>M1A1</p> <p>A1cso (3)</p> <p>M1 A1</p> <p>M1 A1 (4) [7]</p>
	<p>For this question, ignore (a) and (b) labels and award marks wherever correct work is seen.</p> <p>(a) M1 for attempting to use the discriminant of the initial equation (<math>&gt; 0</math> not required, but substitution of <math>a</math>, <math>b</math> and <math>c</math> in the correct formula is required). If the formula <math>b^2 - 4ac</math> is seen, at least 2 of <math>a</math>, <math>b</math> and <math>c</math> must be correct. If the formula <math>b^2 - 4ac</math> is <u>not</u> seen, all 3 (<math>a</math>, <math>b</math> and <math>c</math>) must be correct. This mark can still be scored if substitution in <math>b^2 - 4ac</math> is within the quadratic formula. This mark can also be scored by comparing <math>b^2</math> and <math>4ac</math> (with substitution). However, use of <math>b^2 + 4ac</math> is M0.</p> <p>1<sup>st</sup> A1 for fully correct expression, possibly unsimplified, with <math>&gt;</math> symbol. NB must appear before the last line, even if this is simply in a statement such as <math>b^2 - 4ac &gt; 0</math> or 'discriminant positive'. Condone a bracketing slip, e.g. <math>16 - 4 \times k \times 5 - k</math> if subsequent work is correct and convincing.</p> <p>2<sup>nd</sup> A1 for a fully correct derivation with no incorrect working seen. Condone a bracketing slip if otherwise correct and convincing.</p> <p><u>Using</u> <math>\sqrt{b^2 - 4ac} &gt; 0</math>: Only available mark is the first M1 (unless recovery is seen).</p> <p>(b) 1<sup>st</sup> M1 for attempt to solve an appropriate 3TQ 1<sup>st</sup> A1 for both <math>k = 1</math> and <math>4</math> (only the critical values are required, so accept, e.g. <math>k &gt; 1</math> and <math>k &gt; 4</math>). ** 2<sup>nd</sup> M1 for choosing the "outside" region. A diagram or table alone is not sufficient. Follow through their values of <math>k</math>. The set of values must be 'narrowed down' to score this M mark... listing everything <math>k &lt; 1</math>, <math>1 &lt; k &lt; 4</math>, <math>k &gt; 4</math> is M0.</p> <p>2<sup>nd</sup> A1 for correct answer only, condone "<math>k &lt; 1</math>, <math>k &gt; 4</math>" and even "<math>k &lt; 1</math> and <math>k &gt; 4</math>", but "<math>1 &gt; k &gt; 4</math>" is A0.</p> <p>** Often the statement <math>k &gt; 1</math> and <math>k &gt; 4</math> is followed by the correct final answer. Allow full marks.</p> <p><u>Seeing 1 and 4 used as critical values</u> gives the first M1 A1 by implication.</p> <p>In part (b), condone working with <math>x</math>'s except for the final mark, where the set of values must be a set of values of <math>k</math> (i.e. 3 marks out of 4).</p> <p>Use of <math>\leq</math> (or <math>\geq</math>) in the final answer loses the final mark.</p>	

Question Number	Scheme	Marks
Q6	$b^2 - 4ac$ attempted, in terms of $p$ . $(3p)^2 - 4p = 0$ o.e. Attempt to solve for $p$ e.g. $p(9p - 4) = 0$ Must potentially lead to $p = k, k \neq 0$ $p = \frac{4}{9}$ (Ignore $p = 0$ , if seen)	M1 A1 M1 A1cso [4]
	<p>1<sup>st</sup> M1 for an attempt to substitute into <math>b^2 - 4ac</math> or <math>b^2 = 4ac</math> with <math>b</math> or <math>c</math> correct            Condone <math>x</math>'s in one term only.            This can be inside a square root as part of the quadratic formula for example.  <b>Use of inequalities can score the M marks only</b></p> <p>1<sup>st</sup> A1 for any correct equation: <math>(3p)^2 - 4 \times 1 \times p = 0</math> or better</p> <p>2<sup>nd</sup> M1 for an attempt to factorize or solve their quadratic expression in <math>p</math>.            Method must be sufficient to lead to their <math>p = \frac{4}{9}</math>.</p> <p>Accept factors or use of quadratic formula or <math>(p \pm \frac{2}{9})^2 = k^2</math> (o.e. eg) <math>(3p \pm \frac{2}{3})^2 = k^2</math> or equivalent work on <u>their</u> eqn.</p> $9p^2 = 4p \Rightarrow \frac{9p^{\cancel{2}}}{\cancel{9}} = 4 \text{ which would lead to } 9p = 4 \text{ is OK for this 2}^{\text{nd}} \text{ M1}$ <p>ALT <u>Comparing coefficients</u></p> <p>M1 for <math>(x + \alpha)^2 = x^2 + \alpha^2 + 2\alpha x</math> and A1 for a correct equation eg <math>3p = 2\sqrt{p}</math></p> <p>M1 for forming solving leading to <math>\sqrt{p} = \frac{2}{3}</math> or better</p> <p><u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u>            If the formula is quoted accept some correct substitution leading to a partially correct expression.            If the formula is not quoted only award for a fully correct expression using their values.</p>	

Question number	Scheme	Marks
Q10	<p>(a) <math>(x+2k)^2</math> or <math>\left(x+\frac{4k}{2}\right)^2</math></p> <p><math>(x \pm F)^2 \pm G \pm 3 \pm 11k</math> (where <math>F</math> and <math>G</math> are <u>any</u> functions of <math>k</math>, not involving <math>x</math>)</p> <p><math>(x+2k)^2 - 4k^2 + (3+11k)</math> Accept unsimplified equivalents such as</p> <p><math>\left(x+\frac{4k}{2}\right)^2 - \left(\frac{4k}{2}\right)^2 + 3+11k</math>, <u>and i.s.w. if necessary.</u></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p>
	<p>(b) Accept part (b) solutions seen in part (a).</p> <p>"<math>4k^2 - 11k - 3 = 0</math>"      <math>(4k+1)(k-3) = 0</math>      <math>k = \dots,</math></p> <p>[Or, 'starting again', <math>b^2 - 4ac = (4k)^2 - 4(3+11k)</math> and proceed to <math>k = \dots</math>]</p> <p><math>-\frac{1}{4}</math> and 3      (Ignore any inequalities for the first 2 marks in (b)).</p> <p>Using <math>b^2 - 4ac &lt; 0</math> for no real roots, i.e. "<math>4k^2 - 11k - 3 &lt; 0</math>", to establish inequalities involving their <u>two</u> critical values <math>m</math> and <math>n</math></p> <p>(<u>even if the inequalities are wrong</u>, e.g. <math>k &lt; m, k &lt; n</math>).</p> <p><math>-\frac{1}{4} &lt; k &lt; 3</math> (See conditions below) Follow through their critical values.</p> <p>The final A1ft is still scored if the answer <math>m &lt; k &lt; n</math> follows <math>k &lt; m, k &lt; n</math>.</p> <p><u>Using <math>x</math> instead of <math>k</math> in the final answer</u> loses only the 2<sup>nd</sup> A mark, (condone use of <math>x</math> in earlier working).</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>(4)</p>
	<p>(c) Shape  (seen in (c))</p> <p>Minimum in correct quadrant, <u>not</u> touching the <math>x</math>-axis, <u>not</u> on the <math>y</math>-axis, and there must be no other minimum or maximum.</p> <p>(0, 14) or 14 on <math>y</math>-axis.</p> <p>Allow (14, 0) marked on <math>y</math>-axis.</p> <p>n.b. Minimum is at <math>(-2, 10)</math>, (but there is no mark for this).</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>[10]</p>
	<p>(b) 1<sup>st</sup> M: Forming and solving a 3-term quadratic in <math>k</math> (usual rules.. see general principles at end of scheme). The quadratic must come from "<math>b^2 - 4ac</math>", or from the "<math>q</math>" in part (a).</p> <p>Using <u>wrong discriminant</u>, e.g. "<math>b^2 + 4ac</math>" will score <u>no marks</u> in part (b).</p> <p>2<sup>nd</sup> M: As defined in main scheme above.</p> <p>2<sup>nd</sup> A1ft: <math>m &lt; k &lt; n</math>, where <math>m &lt; n</math>, for their critical values <math>m</math> and <math>n</math>.</p> <p>Other possible forms of the answer (in each case <math>m &lt; n</math>):</p> <p>(i) <math>n &gt; k &gt; m</math></p> <p>(ii) <math>k &gt; m</math> <u>and</u> <math>k &lt; n</math></p> <p>In this case the word "and" must be seen (implying intersection).</p> <p>(iii) <math>k \in (m, n)</math>      (iv) <math>\{k : k &gt; m\} \cap \{k : k &lt; n\}</math></p> <p><u>Not</u> just a number line.</p> <p><u>Not</u> just <math>k &gt; m, k &lt; n</math> (without the word "and").</p> <p>(c) Final B1 is dependent upon a sketch having been attempted in part (c).</p>	

Question Number	Scheme	Marks
4. (a)	$(x+3)^2 + 2$ <p>or <math>p = 3</math> or <math>\frac{6}{2}</math> <math>q = 2</math></p>	B1 B1 (2)
(b)	 <p>U shape with min in 2<sup>nd</sup> quad (Must be above x-axis and not on y-axis)</p> <p>U shape crossing y-axis at (0, 11) only (Condone (11,0) marked on y-axis)</p>	B1 B1 (2)
(c)	$b^2 - 4ac = 6^2 - 4 \times 11$ $= \underline{-8}$	M1 A1 (2) <b>6</b>
<b>Notes</b>		
(a)	Ignore an “= 0” so $(x+3)^2 + 2 = 0$ can score both marks	
(b)	<p>The U shape can be interpreted fairly generously. Penalise an obvious V on 1<sup>st</sup> B1 only. The U needn't have equal “arms” as long as there is a clear min that “holds water”</p> <p>1<sup>st</sup> B1 for U shape with minimum in 2<sup>nd</sup> quad. Curve need not cross the y-axis but minimum should NOT touch x-axis and should be left of (not on) y-axis</p> <p>2<sup>nd</sup> B1 for U shaped curve crossing at (0, 11). Just 11 marked on y-axis is fine. The point must be marked on the sketch (do not allow from a table of values) Condone stopping at (0, 11)</p>	
(c)	<p>M1 for some correct substitution into <math>b^2 - 4ac</math>. This may be as part of the quadratic formula but must be in part (c) and must be only numbers (no x terms present). Substitution into <math>b^2 &lt; 4ac</math> or <math>b^2 = 4ac</math> or <math>b^2 &gt; 4ac</math> is M0</p> <p>A1 for - 8 only. If they write <math>- 8 &lt; 0</math> treat the <math>&lt; 0</math> as ISW and award A1 If they write <math>- 8 \geq 0</math> then score A0 A substitution in the quadratic formula leading to - 8 inside the square root is A0. So substituting into <math>b^2 - 4ac &lt; 0</math> leading to <math>- 8 &lt; 0</math> can score M1A1.</p> <p>Only award marks for use of the discriminant in part (c)</p>	