8 (a)	$(x-\ 6)^2 + (y-\ 4)^2 =\ ;\ 3^2$	B1; B1 (2)
(b)	Complete method for *MP*: $= \sqrt{(12-6)^2 + (6-4)^2}$	M1
	$= \sqrt{40}$ (= 6.325)	A1
	[These first two marks can be scored if seen as part of solution for (c)]	
	Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$	M1
	e.g. $\cos\theta = \frac{\text{MT}}{\text{MP}} = \frac{3}{candidate's\sqrt{40}}$ (= 0.4743) (θ = 61.6835°)	
	[If TP = 6 is used, then M0]	
	θ = 1.0766 rad **AG**	A1 (4)
(c)	Complete method for area *TMP*; e.g. $= \frac{1}{2}\times 3\times\sqrt{40}\sin\theta$	M1
	$= \frac{3}{2}\sqrt{31}$ (= 8.3516..) allow awrt 8.35	A1
	Area (sector) *MTQ* $= 0.5\times 3^2\times 1.0766$ (= 4.8446…)	M1
	Area *TPQ* = candidate's (8.3516.. – 4.8446..)	M1
	= 3.507 awrt [Note: 3.51 is A0]	A1 (5) **[11]**
Notes	(a) Allow 9 for 3^2. (b) First M1 can be implied by √ 40 For second M1: May find TP = $\sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin\theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ (= 0.8803...) *or* $\tan\theta = \frac{\sqrt{31}}{3}$ (1.8859..) or cos rule **NB. Answer is given, but allow final A1 if all previous work is correct.** (c) First M1: (alternative) $\frac{1}{2}\times 3\times\sqrt{40-9}$	

June 2008
Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
5.	(a) $(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1
	$(x\pm3)^2+(y\pm1)^2=k$ or $(x\pm1)^2+(y\pm3)^2=k$ (k a positive value)	M1
	$(x-3)^2+(y-1)^2=29$ (Not $(\sqrt{29})^2$ or 5.39^2)	A1 (4)
	(b) Gradient of radius $=\frac{2}{5}$ (or exact equiv.) Must be seen or used in (b)	B1
	Gradient of tangent $=\frac{-5}{2}$ (Using perpendicular gradient method)	M1
	$y-3=\frac{-5}{2}(x-8)$ (ft gradient of radius, dependent upon both M marks)	M1 A1ft
	$5x+2y-46=0$ (Or equiv., equated to zero, e.g. $92-4y-10x=0$) (Must have integer coefficients)	A1 (5) **9**
	(a) For the M mark, condone one slip inside a bracket, e.g. $(8-3)^2+(3+1)^2$, $(8-1)^2+(1-3)^2$ The first two marks may be gained implicitly from the circle equation. (b) 2nd M: Eqn. of line through (8, 3), in any form, with any grad.(except 0 or ∞). If the 8 and 3 are the 'wrong way round', this M mark is only given if a correct general formula, e.g. $y-y_1=m(x-x_1)$, is quoted. Alternative: 2nd M: Using (8, 3) and an m value in $y=mx+c$ to find a value of c. A1ft: as in main scheme. (Correct substitution of 8 and 3, then a wrong c value will still score the A1ft) (b) Alternatives for the first 2 marks: (but in these 2 cases the 1st A mark is not ft) (i) Finding gradient of tangent by implicit differentiation $2(x-3)+2(y-1)\frac{dy}{dx}=0$ (or equivalent) B1 Subs. $x=8$ and $y=3$ into a 'derived' expression to find a value for dy/dx M1 (ii) Finding gradient of tangent by differentiation of $y=1+\sqrt{20+6x-x^2}$ $\frac{dy}{dx}=\frac{1}{2}(20+6x-x^2)^{-\frac{1}{2}}(6-2x)$ (or equivalent) B1 Subs. $x=8$ into a 'derived' expression to find a value for dy/dx M1 Another alternative: Using $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ $x^2+y^2-6x-2y-19=0$ B1 $8x+3y,\ -3(x+8)-(y+3)-19=0$ M1, M1 A1ft (ft from circle eqn.) $5x+2y-46=0$ A1	

Question Number	Scheme	Marks
5 (a)	PQ: $m_1 = \frac{10-2}{9-(-3)}$ $(=\frac{2}{3})$ **and** QR: $m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1$: $\frac{8}{12} \times \frac{6}{9-a} = -1$ $a = 13$ (*)	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds **all three** of the following	
	$\left(9-(-3)\right)^2 + (10-2)^2$, (*i.e.*208) , $\left(9-a\right)^2 + (10-4)^2$, $\left(a-(-3)\right)^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation	M1
	Solve (or verify) for a, a = 13 (*)	A1 (3)
	(b) Centre is at (5, 3)	B1
	$\left(r^2 =\right)$ $(10-3)^2 + (9-5)^2$ or equiv., or $\left(d^2 =\right)$ $\left(13-(-3)\right)^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown	M1
	Eliminates second unknown	M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3,$ $r^2 = 65$	A1, A1, B1cao (5) **[8]**
Notes (a)	**M1**-considers gradients of *PQ* and *QR* -must be *y* difference / *x* difference (or considers three lengths as in alternative method) **M1** Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) **A1** Obtains a = 13 with no errors by solution or verification. Verification can score 3/3.	
(b)	Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b) **M1** for attempt to find r^2, d^2, r or d (allow one slip in a bracket). **A1** cao. These two marks may be gained implicitly from circle equation **M1** for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. **A1** cao for whole equation and rhs must be 65 or $\left(\sqrt{65}\right)^2$, (similarly B1 must be 65 or $\left(\sqrt{65}\right)^2$, in alternative method for (b))	

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is –3/2 This is **M1** They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9}=-\frac{3}{2}$ **M1** (may still have x in this equation rather than a and there may be a small slip) They then complete to give (a)= 13 **A1** (ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2+7^2=(c+3)^2+1^2$ (equal radii) or $\frac{3-6}{c-3}=-\frac{3}{2}$ **M1** (perpendicular from centre to chord bisects chord) Then using c (= 5) to find a is **M1** Finally a = 13 **A1** (iii) Vector Method: States **PQ. QR** = 0, with vectors stated 12i +8j and (9 – a)**i + 6**j is **M1** Evaluates scalar product so 108 – 12 a + 48 = 0 (**M1**) solves to give a = 13 (**A1**)	M1 M1 A1 M1 M1 A1 M1 M1 A1

Question Number	Scheme	Marks
Q6 (a)	$(x-3)^2-9+(y+2)^2-4=12$ Centre is (3, −2)	M1 A1, A1
	$(x-3)^2+(y+2)^2=12+"9"+"4"$ $r=\sqrt{12+"9"+"4"}=5$ (or $\sqrt{25}$)	M1 A1 (5)
(b)	$PQ=\sqrt{(7--1)^2+(-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
	= 10 = 2×radius, ∴ diam. (N.B. For A1, need a comment or conclusion)	A1 (2)
	[ALT: midpt. of PQ $\left(\frac{7+(-1)}{2},\frac{1+(-5)}{2}\right)$: M1, = (3, −2) = centre: A1]	
	[ALT: eqn. of PQ $3x+4y-1=0$: M1, verify (3, −2) lies on this: A1]	
	[ALT: find two grads, e.g. PQ and P to centre: M1, equal ∴ diameter: A1]	
	[ALT: show that point $S(-1,-5)$ or (7, 1) lies on circle: M1 because $\angle PSQ = 90°$, semicircle ∴ diameter: A1]	
(c)	R must lie on the circle (angle in a semicircle theorem)… often <u>implied</u> by <u>a diagram with R on the circle</u> or by subsequent working)	B1
	$x=0 \Rightarrow$ $y^2+4y-12=0$	M1
	$(y-2)(y+6)=0$ $y=.....$ (M is dependent on previous M)	dM1
	$y=-6$ or 2 (Ignore $y=-6$ if seen, and 'coordinates' are not required))	A1 (4) **[11]**
(a)	1st M1 for attempt to complete square. Allow $(x\pm3)^2\pm k$, or $(y\pm2)^2\pm k$, $k\neq0$. 1st A1 x-coordinate 3, 2nd A1 y-coordinate −2 2nd M1 for a full method leading to $r=$ …, with their 9 and their 4, 3rd A1 5 or $\sqrt{25}$ The 1st M can be <u>implied</u> by $(\pm3, \pm2)$ but a full method must be seen for the 2nd M. Where the 'diameter' in part (b) has <u>clearly</u> been used to answer part (a), no marks in (a), but in this case the M1 (<u>not</u> the A1) for part (b) can be given for work seen in (a). <u>Alternative</u> 1st M1 for comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. 2nd M1 for using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this M mark.	
(c)	1st M1 for setting $x=0$ and getting a 3TQ in y by using eqn. of circle. 2nd M1 (dep.) for attempt to solve a 3TQ leading to <u>at least one</u> solution for y. <u>Alternative 1</u>: (Requires the B mark as in the main scheme) 1st M for using (3, 4, 5) triangle with vertices $(3,-2),(0,-2),(0,y)$ to get a linear or quadratic equation in y (e.g. $3^2+(y+2)^2=25$). 2nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. <u>Alternative 2</u>: (Not requiring realisation that R is on the circle) B1 for attempt at $m_{PR}\times m_{QR}=-1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for setting $x=0$, i.e. $(0, y)$, and proceeding to get a 3TQ in y. Then main scheme. <u>Alternative 2 by 'verification'</u>: B1 for attempt at $m_{PR}\times m_{QR}=-1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for trying (0, 2). 2nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

Question Number	Scheme	Marks
Q8 (a)	$N(2, -1)$	B1, B1 (2)
(b)	$r = \sqrt{\dfrac{169}{4}} = \dfrac{13}{2} = 6.5$	B1 (1)
(c)	Complete Method to find *x* coordinates, $x_2 - x_1 = 12$ and $\dfrac{x_1 + x_2}{2} = 2$ then solve	M1
	To obtain $x_1 = -4, \quad x_2 = 8$	A1ft A1ft
	Complete Method to find *y* coordinates, using equation of circle or Pythagoras i.e. let *d* be the distance below *N* of *A* *then* $d^2 = 6.5^2 - 6^2 \;\Rightarrow\; d = 2.5 \;\Rightarrow y = ..$	M1
	So $y_2 = y_1 = -3.5$	A1 (5)
(d)	Let $A\hat{N}B = 2\theta \;\Rightarrow\; \sin\theta = \dfrac{6}{\text{"6.5"}} \;\Rightarrow\; \theta = (67.38)...$	M1
	So angle *ANB* is 134.8 *	A1 (2)
(e)	*AP* is perpendicular to *AN* so using triangle *ANP* $\tan\theta = \dfrac{AP}{\text{"6.5"}}$	M1
	Therefore *AP* = 15.6	A1cao (2)
		[12]
(a)	B1 for 2 (α) , B1 for –1	
(b)	B1 for 6.5 o.e.	
(c)	1st M1 for finding *x* coordinates – may be awarded if either *x* co-ord is correct A1ft,A1ft are for $\alpha - 6$ and $\alpha + 6$ if *x* coordinate of *N* is α 2nd M1 for a method to find *y* coordinates – may be given if *y* co-ordinate is correct A marks is for –3.5 only.	
(d)	M1 for a full method to find θ or angle *ANB* (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong *y*.** (cos *AN*B = $\dfrac{\text{"6.5"}^2 + \text{"6.5"}^2 - 12^2}{2 \times \text{"6.5"} \times \text{"6.5"}}$ =-0.704) A1 is a printed answer and must be 134.8 – do not accept 134.76.	
(e)	M1 for a full method to find *AP* <u>Alternative Methods</u> N.B. May use triangle *AXP* where *X* is the mid point of *AB*. Or may use triangle ABP. From circle theorems may use angle *BAP* = 67.38 or some variation. Eg $\dfrac{AP}{\sin 67.4} = \dfrac{12}{\sin 45.2}$, $AP = \dfrac{6}{\sin 22.6}$ or $AP = \dfrac{6}{\cos 67.4}$ are each worth M1	

Question Number	Scheme	Marks
10	(a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$	M1 A1
	$(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive value)	M1
	$(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100)	A1
	(Answer only scores full marks)	(4)
	(b) (Gradient of radius =) $\frac{7-1}{10-2}=\frac{6}{8}$ (or equiv.) Must be seen in part (b)	B1
	Gradient of tangent $=\frac{-4}{3}$ (Using perpendicular gradient method)	M1
	$y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞)	M1
	$y-7=\frac{-4}{3}(x-10)$ or equiv (ft gradient of radius, dep. on both M marks)	A1ft
	$\{3y=-4x+61\}$ (N.B. The A1 is only available as ft after B0) The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here. The equation must at some stage be exact, not, e.g. $y=-1.3x+20.3$	
		(4)
	(c) $\sqrt{r^2-\left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag.	M1
	$=\sqrt{10^2-5^2}$ or numerically exact equivalent	A1
	$PQ\ \left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark	A1
		(3) **11**
	(b) 2nd M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞). Alternative: 2nd M: Using (10, 7) and an m value in $y = mx + c$ to find a value of c. (b) Alternative for first 2 marks (differentiation): $2(x-2)+2(y-1)\frac{dy}{dx}=0$ or equiv. B1 Substitute $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$ M1 (This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit'). (c) Alternatives: To score M1, must be a fully correct method to obtain $\frac{1}{2}PQ$ or PQ. 1st A1: For alternative methods that find PQ directly, this mark is for an exact numerically correct version of PQ.	