

Mark Scheme (Results) January 2010

GCE

Core Mathematics C1 (6663)

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Question number	Scheme	Marks
Q1	$x^4 \rightarrow kx^3$ or $x^{1/3} \rightarrow kx^{-2/3}$ or $3 \rightarrow 0$ (k a non-zero constant) $\left(\frac{dy}{dx} = \right) 4x^3$, with '3' differentiated to zero (or 'vanishing') $\left(\frac{dy}{dx} = \right) \dots\dots\dots + \frac{1}{3}x^{-2/3}$ or equivalent, e.g. $\frac{1}{3\sqrt[3]{x^2}}$ or $\frac{1}{3(\sqrt[3]{x})^2}$	M1 A1 A1 [3]
	<p>1st A1 requires $4x^3$, <u>and</u> 3 differentiated to zero. Having '+C' loses the 1st A mark. Terms not added, but otherwise correct, e.g. $4x^3$, $\frac{1}{3}x^{-2/3}$ loses the 2nd A mark.</p>	

Question number	Scheme	Marks
Q4	<p>$x\sqrt{x} = x^{\frac{3}{2}}$ (Seen, or implied by correct integration)</p> <p>$x^{-\frac{1}{2}} \rightarrow kx^{\frac{1}{2}}$ or $x^{\frac{3}{2}} \rightarrow kx^{\frac{5}{2}}$ (k a non-zero constant)</p> <p>$(y =) \frac{5x^{\frac{1}{2}}}{\frac{1}{2}} \dots + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} (+C)$ (“y =” and “+C” are not required for these marks)</p> <p>$35 = \frac{5 \times 4^{\frac{1}{2}}}{\frac{1}{2}} + \frac{4^{\frac{5}{2}}}{\frac{5}{2}} + C$ An equation in C is required (see conditions below).</p> <p>(With their terms simplified or unsimplified).</p> <p>$C = \frac{11}{5}$ or equivalent $2\frac{1}{5}$, 2.2</p> <p>$y = 10x^{\frac{1}{2}} + \frac{2x^{\frac{5}{2}}}{5} + \frac{11}{5}$ (Or equivalent <u>simplified</u>)</p> <p>I.s.w. if necessary, e.g. $y = 10x^{\frac{1}{2}} + \frac{2x^{\frac{5}{2}}}{5} + \frac{11}{5} = 50x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 11$</p> <p>The final A mark requires an <u>equation</u> “y =...” with correct x terms (see below).</p>	<p>B1</p> <p>M1</p> <p>A1... A1</p> <p>M1</p> <p>A1</p> <p>A1 ft</p> <p>[7]</p>
	<p>B mark: $x^{\frac{3}{2}}$ often appears from integration of \sqrt{x}, which is B0.</p> <p>1st A: Any unsimplified or simplified correct form, e.g. $\frac{5\sqrt{x}}{0.5}$.</p> <p>2nd A: Any unsimplified or simplified correct form, e.g. $\frac{x^2\sqrt{x}}{2.5}$, $\frac{2(\sqrt{x})^5}{5}$.</p> <p>2nd M: Attempting to use $x = 4$ <u>and</u> $y = 35$ in a changed function (even if differentiated) to form an equation in C.</p> <p>3rd A: Obtaining $C = \frac{11}{5}$ with no earlier incorrect work.</p> <p>4th A: Follow-through <u>only</u> the value of C (i.e. the other terms must be correct). Accept equivalent <u>simplified</u> terms such as $10\sqrt{x} + 0.4x^2\sqrt{x} \dots$</p>	

Question number	Scheme	Marks
Q6	<p>(a) $y = \frac{x^2 - 5x - 24}{x} = x - 5 - 24x^{-1}$ (or equiv., e.g. $x + 3 - 8 - \frac{24}{x}$)</p> <p>$\frac{dy}{dx} = 1 + 24x^{-2}$ or $\frac{dy}{dx} = 1 + \frac{24}{x^2}$</p> <p>(b) $x = 2: y = -15$ Allow if seen in part (a).</p> <p>$\left(\frac{dy}{dx} = \right) 1 + \frac{24}{4} = 7$ Follow-through from candidate's <u>non-constant</u> $\frac{dy}{dx}$. This must be simplified to a "single value".</p> <p>$y + 15 = 7(x - 2)$ (or equiv., e.g. $y = 7x - 29$) Allow $\frac{y + 15}{x - 2} = 7$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p> <p>B1</p> <p>B1ft</p> <p>M1 A1</p> <p>(4) [8]</p>
	<p>(a) 1st M: Mult. out to get $x^2 + bx + c$, $b \neq 0$, $c \neq 0$ <u>and</u> dividing by x (<u>not</u> x^2). Obtaining one correct term, e.g. $x \dots \dots$ is sufficient evidence of a division attempt.</p> <p>2nd M: <u>Dependent on the 1st M</u>: Evidence of $x^n \rightarrow kx^{n-1}$ for one x term (i.e. not just the constant term) is sufficient). Note that mark is <u>not</u> given if, for example, the numerator and denominator are differentiated separately.</p> <p>A mistake in the 'middle term', e.g. $x + 5 - 24x^{-1}$, does not invalidate the 2nd A mark, so M1 A0 M1 A1 is possible.</p> <p>(b) B1ft: For evaluation, using $x = 2$, of their $\frac{dy}{dx}$, even if unlabelled or called y.</p> <p>M: For the equation, in any form, of a straight line through (2, '-15') with candidate's $\frac{dy}{dx}$ value as gradient.</p> <p>Alternative is to use (2, '-15') in $y = mx + c$ to <u>find a value</u> for c, in which case $y = 7x + c$ leading to $c = -29$ is sufficient for the A1).</p> <p>(See general principles for straight line equations at the end of the scheme). Final A: 'Unsimplified' forms are acceptable, but... $y - (-15) = 7(x - 2)$ is A0 (unresolved 'minus minus').</p>	

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Core Mathematics C2 (6664)

Question Number	Scheme	Marks
Q7	<p>(a) Puts $y = 0$ and attempts to solve quadratic e.g. $(x-4)(x-1) = 0$ Points are (1,0) and (4, 0)</p> <p>(b) $x = 5$ gives $y = 25 - 25 + 4$ and so (5, 4) lies on the curve</p> <p>(c) $\int (x^2 - 5x + 4) dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \quad (+ c)$</p> <p>(d) Area of triangle = $\frac{1}{2} \times 4 \times 4 = 8$ or $\int (x-1) dx = \frac{1}{2}x^2 - x$ with limits 1 and 5 to give 8</p> <p>Area under the curve = $\int_4^5 \left(\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \right) dx = \left[-\frac{5}{6} \right] - \left[-\frac{8}{3} \right]$</p> <p>$\int_4^5 = -\frac{5}{6} - \left(-\frac{8}{3}\right) = \frac{11}{6}$ or equivalent (allow 1.83 or 1.8 here)</p> <p>Area of $R = 8 - \frac{11}{6} = 6\frac{1}{6}$ or $\frac{37}{6}$ or $6.16\bar{r}$ (not 6.17)</p>	<p>M1 A1 (2)</p> <p>B1cso (1)</p> <p>M1A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1 cao</p> <p>A1 cao (5)</p> <p>[10]</p>
	<p>(a) M1 for attempt to find L and M A1 Accept $x = 1$ and $x = 4$, then isw or accept $L = (1,0)$, $M = (4,0)$ Do not accept $L = 1$, $M = 4$ nor $(0, 1)$, $(0, 4)$ (unless subsequent work) Do not need to distinguish L and M. Answers imply M1A1.</p> <p>(b) See substitution, working should be shown, need conclusion which could be just $y = 4$ or a tick. Allow $y = 25 - 25 + 4 = 4$ But not $25 - 25 + 4 = 4$. ($y = 4$ may appear at start) Usually $0 = 0$ or $4 = 4$ is B0</p> <p>(c) M1 for attempt to integrate $x^2 \rightarrow kx^3$, $x \rightarrow kx^2$ or $4 \rightarrow 4x$ A1 for correct integration of all three terms (do not need constant) isw. Mark correct work when seen. So e.g. $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$ is A1 then $2x^3 - 15x^2 + 24x$ would be ignored as subsequent work.</p> <p>(d) B1 for this triangle only (not triangle LMN) 1st M1 for substituting 5 into their changed function 2nd M1 for substituting 4 into their changed function</p>	
	<p>(d) Alternative method: $\int_1^5 (x-1) - (x^2 - 5x + 4) dx + \int_1^4 x^2 - 5x + 4 dx$ can lead to correct answer</p> <p>Constructs $\int_1^5 (x-1) - (x^2 - 5x + 4) dx$ is B1</p> <p>M1 for substituting 5 and 1 and subtracting in first integral M1 for substituting 4 and 1 and subtracting in second integral A1 for answer to first integral i.e. $\frac{32}{3}$ (allow 10.7) and A1 for final answer as before..</p>	

(d)	<p>Another alternative</p> $\int_4^5 (x-1) - (x^2 - 5x + 4) dx + \text{area of triangle } LMP$ <p>Constructs $\int_4^5 (x-1) - (x^2 - 5x + 4) dx$ is B1</p> <p>M1 for substituting 5 and 4 and subtracting in first integral</p> <p>M1 for complete method to find area of triangle (4.5)</p> <p>A1 for answer to first integral i.e. $\frac{5}{3}$ and A1 for final answer as before.</p>
(d)	<p>Could also use</p> $\int_4^5 (4x-16) - (x^2 - 5x + 4) dx + \text{area of triangle } LMN$ <p>Similar scheme to previous one. Triangle has area 6</p> <p>A1 for finding Integral has value $\frac{1}{6}$ and A1 for final answer as before.</p>

Question Number	Scheme	Marks
Q9 (a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10 \right]$ $[y' =] \quad 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ <p>Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$</p> <p>So $x = \frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)</p> <p>$x = 4, \Rightarrow y = 12 \times 2 - 4^{\frac{3}{2}} - 10, \quad \text{so } y = 6$</p>	<p>M1 A1</p> <p>M1</p> <p>M1, A1</p> <p>dM1, A1 (7)</p>
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1)
(a)	<p>1st M1 for an attempt to differentiate a fractional power $x^n \rightarrow x^{n-1}$</p> <p>A1 a.e.f – can be unsimplified</p> <p>2nd M1 for forming a suitable equation using their $y' = 0$</p> <p>3rd M1 for correct processing of fractional powers leading to $x = \dots$ (Can be implied by $x = 4$)</p> <p>A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only.</p> <p>4th M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1</p>	<p>(b) M1 for differentiating their y' again</p> <p>A1 should be simplified</p> <p>(c) B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier).</p> <p>(Treat parts (a),(b) and (c) together for award of marks)</p>