

Mark Scheme (Results) January 2008

GCE

GCE Mathematics (6663/01)

January 2008
6663 Core Mathematics C1
Mark Scheme

| Question number | Scheme | Marks |
|-----------------|--|---|
| 1. | $3x^2 \rightarrow kx^3$ or $4x^5 \rightarrow kx^6$ or $-7 \rightarrow kx$ (k a non-zero constant) $\frac{3x^3}{3}$ or $\frac{4x^6}{6}$ (Either of these, simplified or unsimplified) $x^3 + \frac{2x^6}{3} - 7x$ or equivalent unsimplified, such as $\frac{3x^3}{3} + \frac{4x^6}{6} - 7x^1$ $+ C$ (or any other constant, e.g. $+ K$) | M1 A1 A1 B1 (4) 4 |
| | <p>M: Given for increasing by one the power of x in one of the three terms.</p> <p>A marks: ‘Ignore subsequent working’ after a correct unsimplified version of a term is seen.</p> <p>B: Allow the mark (independently) for an integration constant appearing at any stage (even if it appears, then disappears from the final answer).</p> <p>This B mark can be allowed even when no other marks are scored.</p> | |

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| 5. | <p>(a) $\left(2x^{\frac{1}{2}} + 3x^{-1}\right)$ $p = -\frac{1}{2}, \quad q = -1$</p> <p>(b) $\left(y = 5x - 7 + 2x^{\frac{1}{2}} + 3x^{-1}\right)$</p> <p>$\left(\frac{dy}{dx} = \right) \quad 5 \quad (\text{or } 5x^0) \quad (5x - 7 \text{ correctly differentiated})$</p> <p>Attempt to differentiate either $2x^p$ with a fractional p, giving kx^{p-1} ($k \neq 0$), (the fraction p could be in decimal form) or $3x^q$ with a negative q, giving kx^{q-1} ($k \neq 0$).</p> <p>$\left(-\frac{1}{2} \times 2x^{\frac{3}{2}} - 1 \times 3x^{-2} = \right) \quad -x^{\frac{3}{2}}, -3x^{-2}$</p> | <p>B1, B1 (2)</p> <p>B1</p> <p>M1</p> <p>A1ft, A1ft (4)</p> <p>6</p> |
| | <p>(b):</p> <p>N.B. It is possible to ‘start again’ in (b), so the p and q may be different from those seen in (a), but note that the M mark is for the attempt to differentiate $\underline{\underline{2x^p}}$ or $\underline{\underline{3x^q}}$.</p> <p>However, marks for part (a) <u>cannot</u> be earned in part (b).</p> <p>1st A1ft: ft their $2x^p$, but p must be a fraction and coefficient must be simplified (the fraction p could be in decimal form).</p> <p>2nd A1ft: ft their $3x^q$, but q must be negative and coefficient must be simplified.</p> <p>‘Simplified’ coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +).</p> <p>Having +C loses the B mark.</p> | |

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| 9. | <p>(a) $4x \rightarrow kx^2$ or $6\sqrt{x} \rightarrow kx^{3/2}$ or $\frac{8}{x^2} \rightarrow kx^{-1}$ (k a non-zero constant)</p> <p>$f(x) = 2x^2, -4x^{3/2}, -8x^{-1}$ (+ C) (+ C not required)</p> <p>At $x = 4, y = 1$: $1 = (2 \times 16) - (4 \times 4^{3/2}) - (8 \times 4^{-1}) + C$ <u>Must be in part (a)</u></p> <p>$C = 3$</p> <p>(b) $f'(4) = 16 - (6 \times 2) + \frac{8}{16} = \frac{9}{2}$ ($= m$)</p> <p>Gradient of normal is $-\frac{2}{9}$ ($= -\frac{1}{m}$)</p> <p>Eqn. of normal: $y - 1 = -\frac{2}{9}(x - 4)$ (or any equiv. form, e.g. $\frac{y-1}{x-4} = -\frac{2}{9}$)</p> <p>Typical answers for A1: $\left(y = -\frac{2}{9}x + \frac{17}{9}\right) (2x + 9y - 17 = 0) (y = -0.2\dot{x} + 1.\dot{8})$</p> <p>Final answer: gradient $-\frac{1}{\left(\frac{9}{2}\right)}$ or $-\frac{1}{4.5}$ is A0 (but all M marks are available).</p> | <p>M1</p> <p>A1, A1, A1</p> <p>M1</p> <p>A1 (6)</p> <p>M1</p> <p>M1</p> <p>M1 A1 (4)</p> <p>10</p> |
| | <p>(a) The first 3 A marks are awarded in the order shown, and the terms must be simplified.</p> <p>'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or - sign is allowed (e.g. + - must be replaced by -).</p> <p>2nd M: Using $x = 4$ <u>and</u> $y = 1$ (<u>not</u> $y = 0$) to form an eqn in C. (No C is M0)</p> <p>(b) 2nd M: Dependent upon use of their $f'(x)$.</p> <p>3rd M: eqn. of a straight line through $(4, 1)$ with any gradient except 0 or ∞.</p> <p><u>Alternative for 3rd M</u>: Using $(4, 1)$ in $y = mx + c$ to <u>find a value</u> of c, but an equation (general or specific) must be seen.</p> <p>Having coords the <u>wrong way round</u>, e.g. $y - 4 = -\frac{2}{9}(x - 1)$, loses the 3rd M mark <u>unless</u> a correct general formula is seen, e.g. $y - y_1 = m(x - x_1)$.</p> <p>N.B. The A mark is scored for <u>any</u> form of the correct equation... be prepared to apply isw if necessary.</p> | |

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| 7 | <p>(a) Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing $(6,0)$ (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing $x = 6$]</p> <p>(b) Solving $2x = 6x - x^2$ ($x^2 = 4x$) to $x = ..$ $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,</p> <p>(c) (Area) $= \int_{(0)}^{(4)} (6x - x^2) dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3} (+c)$ Correct use of correct limits on their result above (see notes on limits) $[\frac{3x^2 - x^3}{3}]^4 - [\frac{3x^2 - x^3}{3}]_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$ Area of triangle $= 2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area $= \pm$ (area under curve $-$ area of triangle) applied correctly $(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)</p> | <p>B1 (1)</p> <p>M1 A1 A1 (3)</p> <p>M1 A1 M1 A1 M1 A1 (6)[10]</p> |
| Notes | <p>(b) In scheme first A1: need only give $x = 4$ If <i>verifying approach</i> used: Verifying $(4,8)$ satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, $(0,0)$ needs to be mentioned; accept "clear from diagram"</p> <p>(c) Alternative Using Area $= \pm \int_{(0)}^{(4)} \{(6x - x^2); -2x\} dx$ approach</p> <p>(i) If candidate integrates separately can be marked as main scheme If combine to work with $= \pm \int_{(0)}^{(4)} (4x - x^2) dx$, first M mark and third M mark $= (\pm) [2x^2 - \frac{x^3}{3} (+c)]$ A1, Correct use of correct limits on their result second M1, Totally correct, unsimplified \pm expression (may be implied by correct ans.) A1 $10\frac{2}{3}$ A1 [Allow this if, having given $-10\frac{2}{3}$, they correct it] M1 for correct use of correct limits. Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g. $\pm \{ []^4 - []_0 \}$ If a long method is used, e.g., finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy.</p> <p>Use of trapezium rule: M0A0MA0, possible A1 for triangle M1 (if correct application of trap. rule from $x = 0$ to $x = 4$) A0</p> | |

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| <p>9 (a)</p> <p>(Total area) = $3xy + 2x^2$</p> <p>(Vol:) $x^2y = 100$ $(y = \frac{100}{x^2}, xy = \frac{100}{x})$</p> <p>Deriving expression for area in terms of x only</p> <p>(Substitution, or clear use of, y or xy into expression for area)</p> <p>(Area =) $\frac{300}{x} + 2x^2$ AG</p> <p>(b)</p> <p>$\frac{dA}{dx} = -\frac{300}{x^2} + 4x$</p> <p>Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1</p> <p>[$x^3 = 75$]</p> <p>$x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)</p> <p>(c)</p> <p>$\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4 = \text{positive}$ therefore minimum</p> <p>(d)</p> <p>Substituting found value of x into (a)</p> <p>(Or finding y for found x and substituting both in $3xy + 2x^2$)</p> <p>[$y = \frac{100}{4.2172^2} = 5.6228$]</p> <p>Area = 106.707 awrt 107</p> | <p>B1</p> <p>B1</p> <p>M1</p> <p>A1 cso (4)</p> <p>M1A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>[12]</p> |
| <p>Notes</p> | <p>(a) First B1: Earned for correct unsimplified expression, isw.</p> <p>(c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or “positive”</p> <p>A1: Candidate’s $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion “so minimum”, (allow QED, \checkmark).</p> <p>(may be wrong x, or even no value of x found)</p> <p><u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of “$x = \sqrt[3]{75}$” and consider sign</p> <p>A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum.</p> <p>OR M1: Consider values of A on either side of “$x = \sqrt[3]{75}$” and compare with”107”</p> <p>A1: Both values greater than “$x = 107$” and conclude minimum.</p> <p>Allow marks for (c) and (d) where seen; even if part labelling confused.</p> |