

Binomial Expansions

	$(8a = 10) \quad a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)	A1 (2)
(c)	Substituting their values of a and r into correct formula for sum. $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1) \quad (= 1310718.75) \quad 1\ 310\ 719 \text{ (only this)}$	M1 A1 (2) [6]
Notes:	(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly) (b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^3} \leftarrow \frac{10}{r_c^2} \leftarrow \frac{10}{r_c} \leftarrow 10$ In (a) and (b) correct answer, with no working, allow both marks. (c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula not quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.	
3.	(a)	$\left(1 + \frac{1}{2}x\right)^{10} = 1 + \frac{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}{\underline{\hspace{10em}}}$ $= 1 + 5x; + \frac{45}{4}(\text{or } 11.25)x^2 + 15x^3 \text{ (coeffs need to be these, i.e, simplified)}$ A1; A1 (4) [Allow A1A0, if totally correct with unsimplified, single fraction coefficients]
	(b)	$\left(1 + \frac{1}{2} \times 0.01\right)^{10} = 1 + 5(0.01) + \frac{45}{4} \text{ or } 11.25(0.01)^2 + 15(0.01)^3$ $= 1 + 0.05 + 0.001125 + 0.000015$ $= 1.05114 \quad \text{cao}$
Notes:	(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. (ii) Must have increasing powers of x , (iii) May be listed, need not be added; <i>this applies for all marks.</i> First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, $^{10}C_2$ etc, and condone omission of brackets around powers of $\frac{1}{2}x$ Second A1: Consider as B1 for $1 + 5x$ (b) For M1: Substituting their (0.01) into their (a) result First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)	

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Mark Scheme

Question number	Scheme	Marks
3.	<p>(a) $(1 + ax)^{10} = 1 + 10ax \dots$ (<u>Not</u> unsimplified versions) $+ \frac{10 \times 9}{2}(ax)^2 + \frac{10 \times 9 \times 8}{6}(ax)^3$ Evidence from <u>one</u> of these terms is sufficient $+ 45(ax)^2, + 120(ax)^3$ or $+ 45a^2x^2, + 120a^3x^3$</p> <p>(b) $120a^3 = 2 \times 45a^2$ $a = \frac{3}{4}$ or equiv. (e.g. $\frac{90}{120}, 0.75$) Ignore $a = 0$, if seen</p>	<p>B1</p> <p>M1</p> <p>A1, A1 (4)</p> <p>M1 A1 (2)</p> <p>6</p>
	<p>(a) The terms can be ‘listed’ rather than added.</p> <p>M1: Requires correct structure: ‘binomial coefficient’ (perhaps from Pascal’s triangle) and the correct power of x. (The M mark can also be given for an expansion in <u>descending</u> powers of x). Allow ‘slips’ such as: $\frac{10 \times 9}{2}ax^2, \frac{10 \times 9}{3 \times 2}(ax)^3, \frac{10 \times 9}{2}x^2, \frac{9 \times 8 \times 7}{3 \times 2}a^3x^3$ However, $45 + a^2x^2 + 120 + a^3x^3$ or similar is M0. $\binom{10}{2}$ and $\binom{10}{3}$ or equivalent such as ${}^{10}C_2$ and ${}^{10}C_3$ are acceptable, and even $\binom{10}{2}$ and $\binom{10}{3}$ are acceptable for the method mark.</p> <p>1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (These <u>must</u> be simplified). If simplification is not seen in (a), but correct simplified terms are seen in (b), these marks can be awarded. However, if <u>wrong</u> simplification is seen in (a), this takes precedence.</p> <p><u>Special case:</u> If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost... ... A1 A0 can be given if $45ax^2$ and $120ax^3$ are <u>both</u> achieved.</p> <p>(b) M: Equating their coefficient of x^3 to twice their coefficient of x^2... ... <u>or</u> equating their coefficient of x^2 to twice their coefficient of x^3. (... or coefficients can be <u>correct</u> coefficients rather than their coefficients). Allow this mark even if the equation is trivial, e.g. $120a = 90a$. An equation in a alone is required for this M mark, although... ...condone, e.g. $120a^3x^3 = 90a^2x^2 \Rightarrow (120a^3 = 90a^2 \Rightarrow) a = \frac{3}{4}$.</p> <p><u>Beware:</u> $a = \frac{3}{4}$ following $120a = 90a$, which is A0.</p>	

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Mark Scheme

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1	$(3 - 2x)^5 = 243, \dots + 5 \times (3)^4 (-2x) = -810x \dots$ $+ \frac{5 \times 4}{2} (3)^3 (-2x)^2 = +1080x^2$	B1, B1 M1 A1 (4) [4]
Notes	<p>First term must be 243 for B1, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below).</p> <p>Term must be simplified to $-810x$ for B1</p> <p>The x is required for this mark.</p> <p>The method mark (M1) is generous and is awarded for an attempt at Binomial to get the third term.</p> <p>There must be an x^2 (or no x- i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip).</p> <p>So allow $\binom{5}{2}$ or $\binom{5}{3}$ or 5C_2 or 5C_3 or even $\left(\frac{5}{2}\right)$ or $\left(\frac{5}{3}\right)$ or use of '10' (maybe from Pascal's triangle)</p> <p>May see ${}^5C_2(3)^3(-2x)^2$ or ${}^5C_2(3)^3(-2x^2)$ or ${}^5C_2(3)^5(-\frac{2}{3}x^2)$ or $10(3)^3(2x)^2$ which would each score the M1</p> <p>A1 is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. M1 A1.)</p>	
Special cases	<p>$243 + 810x + 1080x^2$ is B1B0M1A1 (condone no negative signs)</p> <p>Follows correct answer with $27 - 90x + 120x^2$ can isw here (sp case)– full marks for correct answer</p> <p>Misreads <i>ascending</i> and gives $-32x^5 + 240x^4 - 720x^3$ is marked as B1B0M1A0 special case and must be completely correct. (If any slips could get B0B0M1A0)</p> <p>Ignores 3 and expands $(1 \pm 2x)^5$ is 0/4</p> <p>$243, -810x, 1080x^2$ is full marks but $243, -810, 1080$ is B1,B0,M1,A0</p> <p>NB Alternative method $3^5(1 - \frac{2}{3}x)^5 = 3^5 - 5 \times 3^5 \times (\frac{2}{3}x) + \binom{5}{3} 3^5 (-\frac{2}{3}x)^2 + \dots$ is B0B0M1A0</p> <p>– answers must be simplified to $243 - 810x + 1080x^2$ for full marks (awarded as before)</p> <p>Special case $3(1 - \frac{2}{3}x)^5 = 3 - 5 \times 3 \times (\frac{2}{3}x) + \binom{5}{3} 3(-\frac{2}{3}x)^2 + \dots$ is B0, B0, M1, A0</p> <p>Or $3(1 - 2x)^5$ is B0B0M0A0</p>	

Question Number	Scheme	Marks
Q2 (a)	<p>$(7 \times \dots \times x)$ or $(21 \times \dots \times x^2)$ The 7 or 21 can be in ‘unsimplified’ form.</p> $(2 + kx)^7 = 2^7 + 2^6 \times 7 \times kx + 2^5 \times \binom{7}{2} k^2 x^2$ $= 128; + 448kx, + 672k^2 x^2 \text{ [or } 672(kx)^2 \text{]}$ <p>(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)</p>	M1 B1; A1, A1 (4)
(b)	$6 \times 448k = 672k^2$ $k = 4 \quad (\text{Ignore } k = 0, \text{ if seen})$	M1 A1 (2) [6]
(a)	<p>The terms can be ‘listed’ rather than added. Ignore any extra terms.</p> <p>M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing.</p> <p><u>Allow</u> binomial coefficients such as $\binom{7}{1}, \binom{7}{1}, \binom{7}{2}, {}^7C_1, {}^7C_2$.</p> <p>However, $448 + kx$ or similar is M0.</p> <p>B1, A1, A1 for the <u>simplified</u> versions seen above.</p> <p><u>Alternative:</u></p> <p>Note that a factor 2^7 can be taken out first: $2^7 \left(1 + \frac{kx}{2}\right)^7$, but the mark scheme still applies.</p> <p><u>Ignoring subsequent working (isw):</u></p> <p>Isw if necessary after correct working:</p> <p>e.g. $128 + 448kx + 672k^2 x^2$ M1 B1 A1 A1</p> $= 4 + 14kx + 21k^2 x^2 \quad \text{isw}$ <p>(Full marks are still available in part (b)).</p>	
(b)	<p>M1 for equating their coefficient of x^2 to 6 times that of $x \dots$ to get an equation in k, ... <u>or</u> equating their coefficient of x to 6 times that of x^2, to get an equation in k.</p> <p>Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6 \times 448k = 672k$, but beware $k = 4$ following from this, which is A0.</p> <p><u>An equation in k alone</u> is required for this M mark, so...</p> <p>e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow k = 4$ or similar is M0 A0 (equation in coefficients only is never seen), but ...</p> <p>e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow 6 \times 448k = 672k^2 \Rightarrow k = 4$ will get M1 A1 (as coefficients rather than terms have now been considered).</p> <p>The mistake $2 \left(1 + \frac{kx}{2}\right)^7$ would give a maximum of 3 marks: M1B0A0A0, M1A1</p>	

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Mark Scheme

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Q1	$\left[(3-x)^6 = \right] 3^6 + 3^5 \times 6 \times (-x) + 3^4 \times \binom{6}{2} \times (-x)^2$ $= 729, \quad -1458x, \quad +1215x^2$	M1 B1,A1, A1 [4]
Notes	<p>M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form with the correct power of x – condone lack of negative sign and wrong power of 3. This mark may be given if no working is shown, but one of the terms including x is correct. Allow $\frac{6}{1}$, or $\frac{6}{2}$ (must have a power of 3, even if only power 1)</p> <p>First term must be 729 for B1, (writing just 3^6 is B0) can isw if numbers added to this constant later. Can allow 729(1...</p> <p>Term must be simplified to $-1458x$ for A1cao. The x is required for this mark.</p> <p>Final A1 is c.a.o and needs to be $+1215x^2$ (can follow omission of negative sign in working)</p> <p>Descending powers of x would be $x^6 + 3 \times 6 \times (-x)^5 + 3^2 \times \binom{6}{4} \times (-x)^4 + ..$</p> <p>i.e. $x^6 - 18x^5 + 135x^4 + ..$ This is M1B1A0A0 if completely “correct” or M1 B0A0A0 for <u>correct</u> binomial coefficient in any form with the correct power of x as before</p>	
Alternative	<p>NB Alternative method: $(3-x)^6 = 3^6(1+6 \times (-\frac{x}{3}) + \binom{6}{2} \times (-\frac{x}{3})^2 + ..)$ is M1B0A0A0</p> <p>– answers must be simplified to 729, $-1458x$, $+1215x^2$ for full marks (awarded as before)</p> <p>The mistake $(3-x)^6 = 3(1-\frac{x}{3})^6 = 3(1+6 \times (-\frac{x}{3}) + \binom{6}{2} \times (-\frac{x}{3})^2 + ..)$ may also be awarded M1B0A0A0</p> <p>Another mistake $3^6(1-6x+15x^2 \dots) = 729\dots$ would be M1B1A0A0</p>	

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4	<p>(a) $(1 + ax)^7 = 1 + 7ax\dots$ or $1 + 7(ax)\dots$ (<u>Not</u> unsimplified versions)</p> <p>$+ \frac{7 \times 6}{2}(ax)^2 + \frac{7 \times 6 \times 5}{6}(ax)^3$ Evidence from <u>one</u> of these terms is enough</p> <p>$+ 21a^2x^2$ or $+ 21(ax)^2$ or $+ 21(a^2x^2)$</p> <p>$+ 35a^3x^3$ or $+ 35(ax)^3$ or $+ 35(a^3x^3)$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4)</p>
	<p>(b) $21a^2 = 525$</p> <p>$a = \pm 5$ (Both values are required)</p> <p>(The answer $a = 5$ with no working scores M1 A0)</p>	<p>M1</p> <p>A1</p> <p>(2)</p> <p>6</p>
	<p>(a) The terms can be ‘listed’ rather than added.</p> <p>M1: Requires correct structure: a correct binomial coefficient in any form (perhaps from Pascal’s triangle) with the correct power of x. Allow missing a’s and wrong powers of a, e.g.</p> $\frac{7 \times 6}{2}ax^2, \quad \frac{7 \times 6 \times 5}{3 \times 2}x^3$ <p>However, $21 + a^2x^2 + 35 + a^3x^3$ or similar is M0.</p> <p>$1 + 7ax + 21 + a^2x^2 + 35 + a^3x^3 = 57 + \dots$ scores the B1 (isw).</p> <p>$\binom{7}{2}$ and $\binom{7}{3}$ or equivalent such as 7C_2 and 7C_3 are acceptable,</p> <p>but <u>not</u> $\left(\frac{7}{2}\right)$ or $\left(\frac{7}{3}\right)$ (unless subsequently corrected).</p> <p>1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (The binomial coefficients <u>must</u> be simplified).</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p><u>Special case:</u> If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost... ... A1 A0 can be given if $21ax^2$ and $35ax^3$ are <u>both</u> achieved.</p> </div> <p><u>a^2’s omitted throughout:</u> Note that only the M mark is available in this case.</p> <p>(b) M: Equating their coefficient of x^2 to 525.</p> <p>An equation in a or a^2 alone is required for this M mark, but allow ‘recovery’ that shows <u>the required coefficient</u>, e.g.</p> <p>$21a^2x^2 = 525 \Rightarrow 21a^2 = 525$ is acceptable, but $21a^2x^2 = 525 \Rightarrow a^2 = 25$ is not acceptable.</p> <p>After $21ax^2$ in the answer for (a), allow ‘recovery’ of a^2 in (b) so that full marks are available for (b) (but not retrospectively for (a)).</p>	