

BINOMIAL EXPANSIONS 2011-13

Question Number	Scheme	Marks
5.	<p>(a) $\binom{40}{4} = \frac{40!}{4!b!}$; $(1+x)^n$ coefficients of x^4 and x^5 are p and q respectively. $b = 36$ Candidates should usually “identify” two terms as their p and q respectively.</p>	B1 (1)
(b)	<p>Term 1: $\binom{40}{4}$ or ${}^{40}C_4$ or $\frac{40!}{4!36!}$ or $\frac{40(39)(38)(37)}{4!}$ or 91390</p> <p>Term 2: $\binom{40}{5}$ or ${}^{40}C_5$ or $\frac{40!}{5!35!}$ or $\frac{40(39)(38)(37)(36)}{5!}$ or 658008</p> <p>Hence, $\frac{q}{p} = \frac{658008}{91390} \left\{ = \frac{36}{5} = 7.2 \right\}$</p>	<p>Any one of Term 1 or Term 2 correct. (Ignore the label of p and/or q.) M1</p> <p>Both of them correct. (Ignore the label of p and/or q.) A1</p> <p>for $\frac{658008}{91390}$ oe A1 oe cso (3) [4]</p>
Notes		
(a)	B1: for only $b = 36$.	
(b)	<p>The candidate may expand out their binomial series. At this stage no marks should be awarded until they start to identify either one or both of the terms that they want to focus on. Once they identify their terms then if one out of two of them (ignoring which one is p and which one is q) is correct then award M1. If both of the terms are identified correctly (ignoring which one is p and which one is q) then award the first A1.</p> <p>Term 1 = $\binom{40}{4}x^4$ or ${}^{40}C_4(x^4)$ or $\frac{40!}{4!36!}x^4$ or $\frac{40(39)(38)(37)}{4!}x^4$ or $91390x^4$,</p> <p>Term 2 = $\binom{40}{5}x^5$ or ${}^{40}C_5(x^5)$ or $\frac{40!}{5!35!}x^5$ or $\frac{40(39)(38)(37)(36)}{5!}x^5$ or $658008x^5$</p> <p>are fine for any (or both) of the first two marks in part (b).</p> <p>2nd A1 for stating $\frac{q}{p}$ as $\frac{658008}{91390}$ or equivalent. Note that $\frac{q}{p}$ must be independent of x.</p> <p>Also note that $\frac{36}{5}$ or 7.2 or any equivalent fraction is fine for the 2nd A1 mark.</p> <p>SC: If candidate states $\frac{p}{q} = \frac{5}{36}$, then award M1A1A0.</p> <p>Note that either $\frac{4!36!}{5!35!}$ or $\frac{5!35!}{4!36!}$ would be awarded M1A1.</p>	

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2. (a)	$\{(3 + bx)^5\} = (3)^5 + {}^5C_1(3)^4(bx) + {}^5C_2(3)^3(bx)^2 + \dots$ $= 243 + 405bx + 270b^2x^2 + \dots$	243 as a constant term seen. 405bx $({}^5C_1 \times \dots \times x)$ or $({}^5C_2 \times \dots \times x^2)$ $270b^2x^2$ or $270(bx)^2$ B1 B1 <u>M1</u> A1 [4]
(b)	$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$ <p>So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$</p>	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation. $b = 3$ (Ignore $b = 0$, if seen.) M1 A1 [2] 6
(a)	<p>The terms can be “listed” rather than added. Ignore any extra terms. 1st B1: A constant term of 243 seen. Just writing $(3)^5$ is B0. 2nd B1: Term must be simplified to $405bx$ for B1. The x is required for this mark. Note $405 + bx$ is B0. M1: For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 3 and/or b) may be wrong or missing.</p> <p><u>Allow</u> binomial coefficients such as $\binom{5}{2}, \binom{5}{2}, \binom{5}{1}, \binom{5}{1}, {}^5C_2, {}^5C_1$.</p> <p>A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follows $270(bx)^2$, isw and allow A1.)</p> <p><u>Alternative:</u></p> <p>Note that a factor of 3^5 can be taken out first: $3^5\left(1 + \frac{bx}{3}\right)^5$, but the mark scheme still applies.</p> <p><u>Ignore subsequent working (isw):</u> Isw if necessary after correct working: e.g. $243 + 405bx + 270b^2x^2 + \dots$ leading to $9 + 15bx + 10b^2x^2 + \dots$ scores B1B1M1A1 isw. Also note that full marks could also be available in part (b), here.</p> <p><u>Special Case:</u> Candidate writing down the first three terms in descending powers of x usually get $(bx)^5 + {}^5C_4(3)^1(bx)^4 + {}^5C_3(3)^2(bx)^3 + \dots = b^5x^5 + 15b^4x^4 + 90b^3x^3 + \dots$ So award SC: B0B0M1A0 for either $({}^5C_4 \times \dots \times x^4)$ or $({}^5C_3 \times \dots \times x^3)$</p> <p>(b) M1 for equating 2 times their coefficient of x to the coefficient of x^2 to get an equation in b, <u>or</u> equating their coefficient of x to 2 times that of x^2, to get an equation in b. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405b) = 270b$, but beware $b = 3$ from this, which is A0. <u>An equation in b alone is required:</u> e.g. $2(405b)x = 270b^2x^2 \Rightarrow b = 3$ or similar will be Special Case SC: M1A0 (as equation in coefficients only is not seen here). e.g. $2(405b)x = 270b^2x^2 \Rightarrow 2(405b) = 270b^2 \Rightarrow b = 3$ will get M1A1 (as coefficients rather than terms have now been considered). Note: Answer of 3 from no working scores M1A0. Note: The mistake $k\left(1 + \frac{bx}{3}\right)^5, k \neq 243$ would give a maximum of 3 marks: B0B0M1A0, M1A1 Note: For $270bx^2$ in part (a), followed by $2(405b) = 270b^2 \Rightarrow b = 3$, in part (b), allow recovery M1A1.</p>	

Question number	Scheme	Marks
<p>3 (a).</p> <p>(b)</p>	$(1 + \frac{x}{4})^8 = 1 + 2x + \dots$ $+ \frac{8 \times 7}{2} (\frac{x}{4})^2 + \frac{8 \times 7 \times 6}{2 \times 3} (\frac{x}{4})^3,$ $= \quad + \frac{7}{4}x^2 + \frac{7}{8}x^3 \quad \text{or} \quad = \quad + 1.75x^2 + 0.875x^3$ <p>States or implies that $x = 0.1$</p> <p>Substitutes their value of x (provided it is < 1) into series obtained in (a)</p> <p>i.e. $1 + 0.2 + 0.0175 + 0.000875, = 1.2184$</p>	<p>B1</p> <p>M1 A1</p> <p>A1</p> <p>(4)</p> <p>B1</p> <p>M1</p> <p>A1 cao (3)</p> <p style="text-align: right;">7</p>
<p>Alternative for (b) Special case</p>	<p>Starts again and expands $(1 + 0.025)^8$ to</p> $1 + 8 \times 0.025 + \frac{8 \times 7}{2} (0.025)^2 + \frac{8 \times 7 \times 6}{2 \times 3} (0.025)^3, = 1.2184$ <p>(Or $1 + 1/5 + 7/400 + 7/8000 = 1.2184$)</p>	<p>B1,M1,A1</p>
<p>Notes</p>	<p>(a) B1 must be simplified</p> <p>The method mark (M1) is awarded for an attempt at Binomial to get the third and/or fourth term – need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors in powers of 4. Accept any notation for 8C_2 and 8C_3, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs)</p> <p>First A1 is for two completely correct unsimplified terms</p> <p>A1 needs the fully simplified $\frac{7}{4}x^2$ and $\frac{7}{8}x^3$.</p> <p>(b) B1 – states or uses $x = 0.1$ or $\frac{x}{4} = \frac{1}{40}$</p> <p>M1 for substituting their value of x ($0 < x < 1$) into expansion (e.g. 0.1 (correct) or 0.01, 0.00625 or even 0.025 but not 1 nor 1.025 which would earn M0)</p> <p>A1 Should be answer printed cao (not answers which round to) and should follow correct work.</p> <p>Answer with no working at all is B0, M0, A0</p> <p>States 0.1 then just writes down answer is B1 M0A0</p>	

Summer 2012
6664 Core Mathematics 2
Mark Scheme

Question number	Scheme	Marks
1	$[(2-3x)^5] = \dots + \binom{5}{1} 2^4(-3x) + \binom{5}{2} 2^3(-3x)^2 + \dots$ $= 32, -240x, +720x^2$	M1 B1, A1, A1 Total 4
Notes	<p>M1: The method mark is awarded for an attempt at Binomial to get the second and/or third term – need correct binomial coefficient combined with correct power of x. Ignore errors (or omissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for 5C_1 and 5C_2, e.g. $\binom{5}{1}$ and $\binom{5}{2}$ (unsimplified) or 5 and 10 from Pascal’s triangle This mark may be given if no working is shown, but either or both of the terms including x is correct.</p> <p>B1: must be simplified to 32 (writing just 2^5 is B0). 32 must be the only constant term in the final answer- so $32 + 80 - 3x + 80 + 9x^2$ is B0 but may be eligible for M1A0A0 .</p> <p>A1: is cao and is for $-240x$. (not $+240x$) The x is required for this mark</p> <p>A1: is c.a.o and is for $720x^2$ (can follow omission of negative sign in working)</p> <p>A list of correct terms may be given credit i.e. series appearing on different lines</p> <p>Ignore extra terms in x^3 and/or x^4 (isw)</p>	
Special Case	<p>Special Case: <i>Descending powers</i> of x would be</p> $(-3x)^5 + 2 \times 5 \times (-3x)^4 + 2^2 \times \binom{5}{3} \times (-3x)^3 + \dots$ <p>i.e. $-243x^5 + 810x^4 - 1080x^3 + \dots$ This is a misread but award as s.c. M1B1A0A0 if completely “correct” or M1 B0A0A0 for <u>correct binomial coefficient</u> in any form with the correct power of x</p>	
Alternative Method	<p>Method 1: $[(2-3x)^5] = 2^5 \left(1 + \binom{5}{1} \left(-\frac{3x}{2}\right) + \binom{5}{2} \left(\frac{-3x}{2}\right)^2 + \dots \right)$ is M1B0A0A0 { The M1 is for the expression in the bracket and as in first method– need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 3 or sign or bracket errors }</p> <p>– answers must be simplified to $= 32, -240x, +720x^2$ for full marks (awarded as before)</p> $[(2-3x)^5] = 2 \left(1 + \binom{5}{1} \left(-\frac{3x}{2}\right) + \binom{5}{2} \left(\frac{-3x}{2}\right)^2 + \dots \right)$ <p>would also be awarded M1B0A0A0</p> <p>Method 2: Multiplying out : B1 for 32 and M1A1A1 for other terms with M1 awarded if x or x^2 term is correct. Completely correct is 4/4</p>	

January 2013
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1.	$(2 - 5x)^6$	
	$(2^6 =) 64$	Award this when first seen (not $64x^0$)
	$+ 6 \times (2)^5 (-5x) + \frac{6 \times 5}{2} (2)^4 (-5x)^2$	Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $\binom{6}{p} \times (2)^{6-p} (-5x)^p$ with $p = 1$ or $p = 2$ consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients e.g. 6C_1 or $\binom{6}{1}$ or even $\left(\frac{6}{1}\right)$
	$-960x$	Not $+ -960x$
	$(+)6000x^2$	
		(4)
Way 2	$64(1 \pm \dots\dots\dots)$	64 and $(1 \pm \dots\dots -$ Award when first seen.
	$\left(1 - \frac{5x}{2}\right)^6 = 1 - 6 \times \frac{5x}{2} + \frac{6 \times 5}{2} \left(-\frac{5x}{2}\right)^2$	Correct structure for at least one of the underlined terms. E.g. a term of the form: $\binom{6}{p} \times (kx)^p$ with $p = 1$ or $p = 2$ consistently and $k \neq \pm 5$ Condone sign errors. Condone missing brackets if later work implies correct structure but it must be an expansion of $(1 - kx)^6$ where $k \neq \pm 5$
	$-960x$	Not $+ -960x$
	$(+)6000x^2$	
		(4)

Question Number	Scheme	Marks
<p>3. Way 1</p>	$\left(2 - \frac{1}{2}x\right)^8 = 2^8 + \binom{8}{1} \cdot 2^7 \left(-\frac{1}{2}x\right) + \binom{8}{2} 2^6 \left(-\frac{1}{2}x\right)^2 + \binom{8}{3} 2^5 \left(-\frac{1}{2}x\right)^3$ <p>First term of 256</p> $\left({}^8C_1 \times \dots \times x\right) + \left({}^8C_2 \times \dots \times x^2\right) + \left({}^8C_3 \times \dots \times x^3\right)$ $= (256) - 512x + 448x^2 - 224x^3$	<p>B1</p> <p>M1</p> <p>A1, A1</p> <p>(4)</p> <p>Total 4</p>
<p>Way 2</p>	$\left(2 - \frac{1}{2}x\right)^8 = 2^8 \left(1 - \frac{1}{4}x\right)^8 = 2^8 \left(1 + \binom{8}{1} \cdot \left(-\frac{1}{4}x\right) + \binom{8}{2} \left(-\frac{1}{4}x\right)^2 + \binom{8}{3} \left(-\frac{1}{4}x\right)^3\right)$ <p>Scheme is applied exactly as before except in special case below*</p>	
Notes for Question 3		
<p>B1: The first term should be 256 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept 8C_1 or $\binom{8}{1}$ or 8 as a coefficient, and 8C_2 or $\binom{8}{2}$ or 28 as another..... Pascal's triangle may be used to establish coefficients. A1: Any two of the final three terms correct (but allow +- instead of -) A1: All three of the final three terms correct and simplified. (Deduct last mark for $+512x$ and $+224x^3$ in the series). Also deduct last mark for the three terms correct but unsimplified. (Accept answers without + signs, can be listed with commas or appear on separate lines) The common error $\left(2 - \frac{1}{2}x\right)^8 = 256 + \binom{8}{1} \cdot 2^7 \left(-\frac{1}{2}x\right) + \binom{8}{2} 2^6 \left(-\frac{1}{2}x\right)^2 + \binom{8}{3} 2^5 \left(-\frac{1}{2}x\right)^3$ would earn B1, M1, A0, A0 Ignore extra terms involving higher powers. Condone terms in reverse order i.e. $= -224x^3 + 448x^2 - 512x + (256)$ *In Way 2 the error $= 2 \left(1 + \binom{8}{1} \cdot \left(-\frac{1}{4}x\right) + \binom{8}{2} \left(-\frac{1}{4}x\right)^2 + \binom{8}{3} \left(-\frac{1}{4}x\right)^3\right)$ giving $= 2 - 4x + \frac{7}{2}x^2 - \frac{7}{4}x^3$ is a special case B0, M1, A1, A0 i.e. 2/4</p>		