

## Normal Distribution Mark Scheme 2011-13

Question Number	Scheme	Marks
8.		
(a)	$P(X > 168) = P\left(Z > \frac{168-160}{5}\right)$ $= P(Z > 1.6)$ $= 0.0548$ <p style="text-align: right;"><b>awrt 0.0548</b></p>	M1 A1 A1 (3)
(b)	$P(X < w) = P\left(Z < \frac{w-160}{5}\right)$ $\frac{w-160}{5} = -2.3263$ $w = 148.37$ <p style="text-align: right;"><b>awrt 148</b></p>	M1 B1 A1 (3)
(c)	$\frac{160 - \mu}{\sigma} = 2.3263$ $\frac{152 - \mu}{\sigma} = -1.2816$ $160 - \mu = 2.3263\sigma$ $152 - \mu = -1.2816\sigma$ $8 = 3.6079\sigma$ $\sigma = 2.21\dots$ $\mu = 154.84\dots$ <p style="text-align: right;"><b>awrt 2.22</b> <b>awrt 155</b></p>	M1 B1 B1 M1 A1 A1 (6) [12]
<b>Notes</b>		
(a)	M1 for an attempt to standardize 168 with 160 and 5 i.e. $\pm\left(\frac{168-160}{5}\right)$ or implied by 1.6 1 <sup>st</sup> A1 for $P(Z > 1.6)$ or $P(Z < -1.6)$ ie $z = 1.6$ and a correct inequality or 1.6 on a shaded diagram <p style="text-align: center;"><b>Correct answer to (a) implies all 3 marks</b></p>	
(b)	M1 for attempting $\pm\left(\frac{w-160}{5}\right) =$ recognizable $z$ value ( $ z  > 1$ ) B1 for $z = \pm 2.3263$ or better. Should be $z = \dots$ or implied so: $1 - 2.3263 = \frac{w-160}{5}$ is M0B0 A1 for awrt 148. This may be scored for other $z$ values so M1B0A1 is possible <p style="text-align: center;"><b>For awrt 148 only with no working seen award M1B0A1</b></p>	
(c)	M1 for attempting to standardize 160 or 152 with $\mu$ and $\sigma$ (allow $\pm$ ) <u>and</u> equate to $z$ value ( $ z  > 1$ ) 1 <sup>st</sup> B1 for awrt $\pm 2.33$ or $\pm 2.32$ seen 2 <sup>nd</sup> B1 for awrt $\pm 1.28$ seen 2 <sup>nd</sup> M1 for attempt to solve their two linear equations in $\mu$ and $\sigma$ leading to equation in just one variable 1 <sup>st</sup> A1 for $\sigma =$ awrt 2.22 . Award when 1 <sup>st</sup> seen 2 <sup>nd</sup> A1 for $\mu =$ awrt 155. Correct answer only for part (c) can score all 6 marks. NB $\sigma = 2.21$ commonly comes from $z = 2.34$ and usually scores M1B0B1M1A0A1 <p style="text-align: center;"><b>The A marks in (c) require both M marks to have been earned</b></p>	

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<b>2.</b> <b>(a)</b>	$\frac{23 - \mu}{5} = "1.40" \quad (\text{o.e})$ $\frac{\mu = 16}{16.0}$	awrt $\pm 1.40$ B1 M1A1ft (or awrt A1 (4)
<b>(b)</b>	<u>0.4192</u>	B1 (1)
<b>Notes</b>		
<b>(a)</b>	B1 for awrt $\pm 1.40$ or better seen anywhere. Condone 1.4 instead of 1.40 M1 for attempting to standardise with 23 and 5 and $\mu$ , accept $\pm$ e.g. $\frac{23 - \mu}{25} = 1.40$ can score B1M0 (since using 25 not 5 for standardising) $\frac{23 - \mu}{5} = 0.9192$ can score B0M1 (since have correct standardisation) Can accept equivalent equations e.g. $23 - \mu = 5 \times "1.40"$ 1 <sup>st</sup> A1ft for standardised expression = to a z value ( $ z  > 1$ ). Signs must be compatible. Follow through their z e.g. $\frac{23 - \mu}{5} = \text{their } z \text{ where } z > 1$ or $\frac{\mu - 23}{5} = \text{their } z \text{ where } z < -1$ 2 <sup>nd</sup> A1 for 16 or awrt 16.0 if they are using a more accurate z <b>Correct answer only scores 4/4 but if any working is seen apply scheme</b>	
<b>(b)</b>	B1 for 0.4192 (but accept 3sf accuracy if 0.9192 – 0.5 is seen)	

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<p><b>4.</b></p> <p><b>(a)</b></p> <p><b>(b)</b></p>	$(z = \pm) \frac{15 - 16.12}{1.6} (= -0.70)$ $P(Z < -0.70) = 1 - 0.7580$ $= \underline{0.2420} \quad \text{(awrt 0.242)}$ <p>[P(T &lt; t) = 0.30 implies] <math>z = \frac{t - 16.12}{1.6} = -0.5244</math></p> $\frac{t - 16.12}{1.6} = -0.5244 \Rightarrow t = 16.12 - 1.6 \times "0.5244"$ $t = \text{awrt } \underline{15.28} \text{ (allow awrt 15.28/9)}$	<p>M1</p> <p>M1 A1</p> <p>(3)</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>(4) 7</p>
<b>Notes</b>		
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p style="text-align: center;"><b>Allow slips e.g. 16.2 for 16.12 for 1<sup>st</sup> M1 in (a) and (b)</b></p> <p>1<sup>st</sup> M1 for standardising expression with 15, 16.12 and 1.6 - allow <math>\pm</math>  2<sup>nd</sup> M1 for 1 - a probability (&gt; 0.5) from tables or calculator based on their standardised value</p> <p style="text-align: center;"><b>Correct answer only scores 3/3</b></p> <p style="text-align: center;"><b>In part (b) they can use any letter or symbol instead of t</b></p> <p>1<sup>st</sup> M1 for standardising with t (o.e.), 16.12 and 1.6, allow <math>\pm</math>, and setting equal to a z value  1<sup>st</sup> A1 for an equation with <math>z = \pm 0.5244</math> or better  e.g. <math>\frac{t - 16.12}{1.6} = \pm 0.52</math> (or 0.525) scores M1 (but A0)</p> <p>2<sup>nd</sup> M1 for solving <u>their</u> linear equation as far as <math>t = a \pm b \times 1.6</math>. Not dependent on 1<sup>st</sup> M1  e.g. solving <math>\frac{t - 16.12}{1.6} = 0.3</math> to give <math>t = 16.12 + 1.6 \times 0.3</math> scores this M1</p> <p>Allow <math>\frac{t - 16.12}{1.6^2} = 0.3</math> to give <math>t = 16.12 + 1.6^2 \times 0.3</math> to score M1 too</p> <p>2<sup>nd</sup> A1 dependent on both M marks. Allow awrt 15.28 or awrt 15.29  Condone awrt 15.3 if a correct expression for <math>t = \dots</math> is seen.</p> <p><b>Answers with no working:</b>  15.28 is M1A1M1A1, 15.29 is M1A0M1A1, 15.3 is M1A0M1A0</p>	

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<p><b>7 (a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p>	$P(W < 224) = P\left(z < \frac{224 - 232}{5}\right)$ $= P(z < -1.6)$ $= 1 - 0.9452$ $= 0.0548$ $0.5 - 0.2 = 0.3$ $\frac{w - 232}{5} = 0.5244$ $w = 234.622$ $0.2 \times (1 - 0.2)$ $2 \times 0.8 \times (1 - 0.8) = 0.32$	<p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>M1</p> <p>0.3 or 0.7 seen</p> <p>M1</p> <p>0.5244 seen</p> <p>B1; M1</p> <p>A1</p> <p>(4)</p> <p>M1</p> <p>M1 A1</p> <p>(3)</p> <p><b>Total 10</b></p>
<p><b>NOTES</b></p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>M1 for standardising with 232 and 5. (i.e. not <math>5^2</math> or <math>\sqrt{5}</math>). Accept <math>\pm \frac{w - 232}{5}</math>.</p> <p>M1 for finding (1 - a probability &gt; 0.5)</p> <p>A1 awrt 0.0548</p> <p>M1 Can be implied by use of <math>\pm 0.5244</math> or <math>\pm (0.52 \text{ to } 0.53)</math></p> <p>B1 for <math>\pm 0.5244</math> only.</p> <p>Second M1 standardise with 232 and 5 and equate to <math>z</math> value of (0.52 to 0.53) or (0.84 to 0.85)</p> <p>1 - <math>z</math> used award second M0.</p> <p>Require consistent signs i.e. <math>\frac{232 - w}{5} = -0.5244</math> or negative <math>z</math> value for M1.</p> <p>A1 dependent upon second M mark for awrt 235 but see note below.</p> <p>Common errors involving probabilities and not <math>z</math> values:</p> <p><math>P(Z &lt; 0.2) = 0.5793</math> used instead of <math>z</math> value gives awrt 235 but award M0B0M0A0</p> <p><math>P(Z &lt; 0.8) = 0.7881</math> used instead of <math>z</math> value award M0B0M0A0.</p> <p>M1B0M0A0 for 0.6179, M1B0M0A0 for 0.7580</p> <p>M1 for 0.16 seen</p> <p>M1 for '<math>2 \times p(1 - p)</math>'</p> <p>A1 0.32 correct answer only</p>	

Question	Scheme	Marks
<p>6. (a)</p> <p>(b)</p> <p>(c)</p>	$[z =] \pm \left( \frac{150 - 162}{7.5} \right)$ $[z =] - 1.6$ $[P(F > 150) = P(Z > -1.6) =] = 0.9452(0071\dots)$ $z = \pm 0.2533 \text{ (or better seen)}$ $(\pm) \frac{s - 162}{7.5} = 0.2533(47\dots)$ $s = 163.9$ $z = \pm 1.2816 \text{ (or better seen)}$ $\frac{162 - \mu}{9} = -1.2815515\dots$ $\mu = 173.533\dots$	<p>M1</p> <p>A1</p> <p>A1 (3) <b>awrt 0.945</b></p> <p>B1</p> <p>M1</p> <p>A1 (3) <b>awrt 164</b></p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1 (4) <b>awrt 174</b></p> <p style="text-align: right;"><b>[10]</b></p>
<b>Notes</b>		
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p><b>NB</b></p>	<p>M1 for attempting to standardise with 150, 162 and 7.5. Accept <math>\pm</math>  Allow use of symmetry and therefore 174 instead of 150  1<sup>st</sup> A1 for <math>-1.6</math> seen. Allow 1.6 seen if 174 used or awrt 0.945 is seen. Sight of 0.945(2) is A1.  2<sup>nd</sup> A1 for awrt 0.945 Do not apply ISW, if 0.9452 is followed by <math>1 - 0.9452</math> then award A0  Correct answer only 3/3</p> <p>B1 for <math>(z =) \pm 0.2533</math> (or better) seen.  Giving <math>z = \pm 0.25</math> or <math>\pm 0.253</math> scores B0 here but may get M1A1  M1 for standardising with <math>s</math> (o.e.), 162 and 7.5, allow <math>\pm</math>, and setting equal to a <math>z</math> value  Only allow <math>0.24 \leq z \leq 0.26</math> Condone e.g. 160 for 162 etc  A1 for awrt 164 (Correct answer only scores B0M1A1)</p> <p>B1 for <math>(z =) \pm 1.2816</math> (or better) seen. Allow awrt <math>\pm 1.28</math> if B0 scored in (b) for <math>z = \text{awrt} \pm 0.25</math>  M1 for attempting to standardise with 162, 9 and <math>\mu</math>, and setting equal to a <math>z</math> value where  <math>1.26 &lt;  z  &lt; 1.31</math>. Allow <math>\pm</math> here so signs don't have to be compatible.  1<sup>st</sup> A1 for a correct equation <u>with</u> compatible signs and <math>1.26 &lt;  z  &lt; 1.31</math>  2<sup>nd</sup> A1 for awrt 174 (Correct answer only scores B0M1A1A1). <b>Dependent on 1<sup>st</sup> A1</b></p> <p>An equation <math>\frac{162 - \mu}{9} = 1.2816</math> leading to an answer of <math>\mu = 174</math> is A0A0 <u>unless</u> there is clear  correct working such as: <math>\frac{162 - x}{9} = 1.2816 \Rightarrow x = \dots \therefore \mu = 162 + (162 - x) = 174</math> then award A1A1</p> <p>A common error is: <math>\frac{162 - \mu}{9} = 1.2816</math> followed by <math>\mu = 162 + 9 \times 1.2816 = \text{awrt } 174</math> It gets  A0A0</p>	

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<p><b>4.</b></p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p><b>S.C.</b></p>	$\frac{127-100}{15}$ <p>So <math>P(L &gt; 127) = P(Z &gt; 1.8)</math> or <math>1 - P(Z &lt; 1.8)</math> o.e.  <math>= 1 - 0.9641 = \underline{\mathbf{0.0359}}</math> (awrt <b>0.0359</b>)</p> $\frac{d-100}{15} = -1.2816 \quad (\text{Calculator gives } -1.2815515\dots)$ $d = 80.776 \quad (\text{awrt } \underline{\mathbf{80.8}})$ <p>Require <math>P(L &gt; 133   L &gt; 127)</math></p> $= \frac{P(L > 133)}{P(L > 127)} = \frac{P(Z > 2.2)}{P(L > 127)}$ $= \frac{1-0.9861}{1-0.9641} = \frac{0.0139}{0.0359}$ $= 0.3871\dots = \text{awrt } \underline{\mathbf{0.39}}$ <p>An attempt at <math>P(L &lt; 133   L &gt; 127)</math> that leads to awrt 0.61 (M0M1A0A0)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>M1, B1</p> <p>A1</p> <p>(3)</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>A1</p> <p>(4)</p> <p><b>10</b></p>
<b>Notes</b>		
<p>(a)</p> <p>(b)</p> <p><b>Calc</b></p> <p>(c)</p> <p><b>ALT</b></p>	<p>M1 for attempting to standardise with 127, 100 and 15. Allow <math>\pm</math></p> <p>1<sup>st</sup> A1 for <math>Z &gt; 1.8</math>. Allow a diagram but must have 1.8 and correct area indicated. Must have the Z so <math>P(L &gt; 127)</math> with or without a diagram is insufficient. May be implied by 0.0359</p> <p>2<sup>nd</sup> A1 for awrt 0.0359 (calc. gives 0.035930266...). Correct ans only 3/3. M1A0A1 not poss.</p> <p>M1 for an attempt to standardise with 100 and 15 and set = <math>\pm</math> any z value (<math> z  &gt; 1</math>)</p> <p>B1 for <math>z = \pm 1.2816</math> (or better) seen anywhere [May be implied by 80.776(72...) or better seen]</p> <p>A1 for awrt 80.8 (can be scored for using 1.28 but then they get M1B0A1)</p> <p>The 80.8 must follow from correct working.</p> <p>If answer is awrt 80.8 <b>and</b> awrt 80.777 or 80.776... or better seen then award M1B1A1</p> <p>If answer is awrt 80.8 or 80.77 then award M1B0A1 (unless of course <math>z = 1.2816</math> is seen)</p> <p>1<sup>st</sup> M1 for clear indication of correct conditional probability or attempt at correct ratio</p> <p>So clear attempt at <math>\frac{P(L &gt; 133)}{P(L &gt; 127)}</math> is sufficient for the 1<sup>st</sup> M1</p> <p>2<sup>nd</sup> dM1 dependent on 1<sup>st</sup> M1 for <math>P(L &gt; 133)</math> leading to <math>P(Z &gt; 2.2)</math>.</p> <p>1<sup>st</sup> A1 for 0.0139 or better seen coming from <math>P(Z &gt; 2.20)</math>. Dependent on both Ms</p> <p>2<sup>nd</sup> A1 for awrt 0.39. Both Ms required</p> <p>If they assume Alice did not check that the phone was working you may see:  <math>[P(L &lt; 127).0] + P(L &gt; 127).P(L &gt; 133   L &gt; 127)</math> Provided the <u>conditional probability</u> is seen as part of this calculation the 1<sup>st</sup> M1 can be scored and their final answer will be 0.0139(4/4)  An answer of 0.0139 without sight of the conditional probability is 0/4.</p>	

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<p>6. (a)</p> <p>(b)</p> <p>(c)</p>	<p>[Let <math>X</math> be the amount of beans in a tin. <math>P(X &lt; 200) = 0.1</math>]</p> $\frac{200 - \mu}{7.8} = -1.2816$ <p>[ calc gives 1.28155156...]</p> $\mu = 209.996\dots$ <p>awrt 210</p> <p>(b)</p> $P(X > 225) = P\left(Z > \frac{225 - "210"}{7.8}\right)$ $= P(Z > 1.92) \text{ or } 1 - P(Z < 1.92) \quad (\text{allow } 1.93)$ $= 1 - 0.9726 = 0.0274 \text{ (or better) [calc gives 0.0272037...]}$ $= 0.0274$ <p>= awrt <b>2.7%</b> allow <b>0.027</b></p> <p>(c)</p> <p>[Let <math>Y</math> be the new amount of beans in a tin]</p> $\frac{210 - 205}{\sigma} = 2.3263 \quad \text{or} \quad \frac{200 - 205}{\sigma} = -2.3263 \quad [\text{calc gives 2.3263478...}]$ $\sigma = \frac{5}{2.3263}$ $\sigma = 2.15 \text{ (2.14933...)}$	<p>M1 B1</p> <p>A1</p> <p>(3)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>M1 B1</p> <p>dM1</p> <p>A1</p> <p>(4)</p> <p>(10 marks)</p>
<b>Notes</b>		
	<p><b>Condone poor handling of notation if answers are correct but A marks must have correct working.</b></p> <p>(a) M1 for an attempt to standardise (allow <math>\pm</math>) with 200 and 7.8 and set = <math>\pm</math> any <math>z</math> value (<math> z  &gt; 1</math>)  B1 for <math>z = \pm 1.2816</math> (or better used as a <math>z</math>) [May be implied by 209.996(102...) or better seen]  A1 for awrt 210 (can be scored for using 1.28 but then they get M1B0A1)  The 210 must follow from correct working – sign scores A0  If answer is awrt 210 <b>and</b> 209.996... or better seen then award M1B1A1  <math>z = 1.28</math> gives 209.984 and <math>z = 1.282</math> gives 209.9996 and both score M1B0A1  If answer is awrt 210 or awrt 209.996 then award M1B0A1 (unless of course <math>z = 1.2816</math> is seen)</p> <p>(b) M1 for attempting to standardise with 225, their mean and 7.8. Allow <math>\pm</math>  1<sup>st</sup> A1 for <math>Z &gt;</math> awrt 1.92/3. Allow a diagram but must have 1.92/3 and correct area indicated.  Must have the <math>Z</math> so <math>P(X &gt; 225)</math> with or without a diagram is not sufficient.  Award for <math>1 - 0.9726</math> or <math>1 - 0.9732</math>  2<sup>nd</sup> A1 for 2.7 % or better (calculator gives 2.72...) Allow awrt 0.027. Correct ans scores 3/3</p> <p>(c) 1<sup>st</sup> M1 for an attempt to standardise with 200 or 210, 205 and <math>\sigma</math> and set = <math>\pm</math> any <math>z</math> value (<math> z  &gt; 2</math>)  B1 for <math>z = 2.3263</math> (or better) <b>and</b> compatible signs.  If B0 in (a) for using a value in [1.28, 1.29) but not using 1.2816: allow awrt 2.33 here  2<sup>nd</sup> dM1 <b>Dependent on the first M1</b> for correctly rearranging to make <math>\sigma = \dots</math> May be implied  e.g. <math>\frac{5}{\sigma} = 2.32 \rightarrow \sigma = 2.16</math> (M1A0) BUT must have <math>\sigma &gt; 0</math>  A1 for awrt 2.15. Must follow from correct working but a range of possible <math>z</math> values will do.  NB <math>2.320 &lt; z \leq 2.331</math> will give an answer of awrt 2.15</p>	