

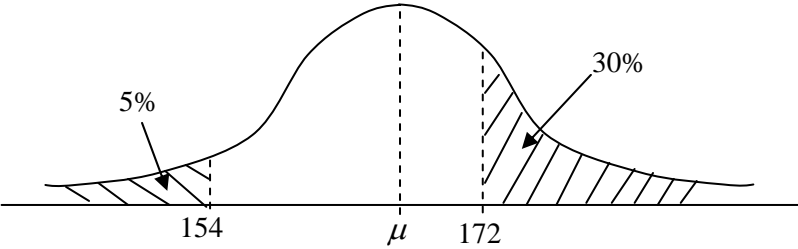
## Normal Distribution 2008-10

6.			
(a)		200 or 200g	<b>B1</b>
(b)		$P(190 < X < 210) = 0.6$ or $P(X < 210) = 0.8$ or $P(X > 210) = 0.2$ or diagram (o.e.) Correct use of 0.8 or 0.2  $Z = (\pm) \frac{210 - 200}{\sigma}$ $\frac{10}{\sigma} = 0.8416$ $\sigma = 11.882129\dots$	<b>(1)</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>B1</b> <b>A1</b> <b>M1</b> <b>A1</b>
(c)		$P(X < 180) = P\left(Z < \frac{180 - 200}{\sigma}\right)$ $= P(Z < -1.6832)$ $= 1 - 0.9535$ $= 0.0465 \text{ or AWRT } 0.046$	<b>(5)</b> <b>M1</b> <b>M1</b> <b>A1</b> <b>(3)</b> <b>Total 9 marks</b>
(a)		<p>“mean = 200g” is B0 but “median = 200” or just “200” alone is B1</p> <p><b>Standardization in (b) and (c).</b> They must use <math>\sigma</math> not <math>\sigma^2</math> or <math>\sqrt{\sigma}</math>.</p>	
(b)	1 <sup>st</sup> M1	for a correct probability statement (as given or eg $P(200 < X < 210) = 0.3$ o.e.) or shaded diagram - must have values on z-axis and probability areas shown	
	1 <sup>st</sup> A1	for correct use of 0.8 or $p = 0.2$ . Need a correct probability statement. May be implied by a suitable value for z seen (e.g. $z = 0.84$ )	
	2 <sup>nd</sup> M1	for attempting to standardise. Values for $x$ and $\mu$ used in formula. Don't need $z =$ for this M1 nor a z-value, just mark standardization.	
	B1	for $z = 0.8416$ (or better) [ $z = 0.84$ usually just loses this mark in (a)]	
	2 <sup>nd</sup> A1	for AWRT 11.9	
(c)	1 <sup>st</sup> M1	for attempting to Standardise with 200 and their sd(>0) e.g. $(\pm) \frac{180 - 200}{\text{their } \sigma}$	
	2 <sup>nd</sup> M1	<b>NB on open this is an A mark ignore and treat it as 2<sup>nd</sup> M1</b> for 1 - a probability from tables provided compatible with their probability statement.	
	A1	for 0.0465 or AWRT 0.046 (Dependent on both Ms in part (c))	

Question Number	Scheme	Marks
<p>Q7 (a)</p> <p>(b)</p> <p>(c)</p>	$z = \frac{53 - 50}{2}$ <p>Attempt to standardise</p> <p>1-probability required can be implied</p> $P(X > 53) = 1 - P(Z < 1.5)$ $= 1 - 0.9332$ $= 0.0668$ $P(X \leq x_0) = 0.01$ $\frac{x_0 - 50}{2} = -2.3263$ $x_0 = 45.3474$ <p>awrt 45.3 or 45.4</p> $P(2 \text{ weigh more than } 53\text{kg and } 1 \text{ less}) = 3 \times 0.0668^2 (1 - 0.0668)$ $= 0.012492487..$ <p>awrt 0.012</p>	<p><b>M1</b></p> <p><b>B1</b></p> <p><b>A1</b></p> <p>[3]</p> <p><b>M1</b></p> <p><b>M1B1</b></p> <p><b>M1A1</b></p> <p>[5]</p> <p><b>B1M1A1ft</b></p> <p><b>A1</b></p> <p>[4]</p> <p><b>Total 12</b></p>
	<p><u>Notes:</u></p> <p>(a) M1 for using 53,50 and 2, either way around on numerator  B1 1- any probability for mark  A1 0.0668 cao</p> <p>(b) M1 can be implied or seen in a diagram  or equivalent with correct use of 0.01 or 0.99  M1 for attempt to standardise with 50 and 2 numerator either way around  B1 for <math>\pm 2.3263</math>  M1 Equate expression with 50 and 2 to a <math>z</math> value to form an equation with consistent signs and attempt to solve  A1 awrt 45.3 or 45.4</p> <p>(c) B1 for 3,  M1 <math>p^2(1 - p)</math> for any value of <math>p</math>  A1ft for <math>p</math> is their answer to part (a) without 3  A1 awrt 0.012 or 0.0125</p>	

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6	<p>(a) <math>P(X &lt; 39) = P\left(Z &lt; \frac{39-30}{5}\right)</math>  <math>= P(Z &lt; 1.8) = \underline{0.9641}</math> (allow awrt 0.964)</p> <p>(b) <math>P(X &lt; d) = P\left(Z &lt; \frac{d-30}{5}\right) = 0.1151</math>  <math>1 - 0.1151 = 0.8849</math> (allow <math>\pm 1.2</math>)  <math>\Rightarrow z = -1.2</math>  <math>\therefore \frac{d-30}{5} = -1.2</math> <math>\underline{d = 24}</math></p> <p>(c) <math>P(X &gt; e) = 0.1151</math> so <math>e = \mu + (\mu - \text{their } d)</math> or <math>\frac{e-30}{5} = 1.2</math> or <math>-</math> their <math>z</math>  <math>\underline{e = 36}</math></p> <p>(d) <math>P(d &lt; X &lt; e) = 1 - 2 \times 0.1151</math>  <math>= 0.7698</math> AWRT <math>\underline{0.770}</math></p>	<p>M1 A1 (2)</p> <p>M1 B1 M1A1 (4)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>[10]</p>
	<p><b>Answer only scores all marks in each section BUT check (b) and (c) are in correct order</b></p> <p>(a) M1 for standardising with <math>\sigma</math>, <math>z = \pm \frac{39-30}{5}</math> is OK  A1 for 0.9641 or awrt 0.964 but if they go on to calculate <math>1 - 0.9641</math> they get M1A0</p> <p>(b) 1<sup>st</sup> M1 for attempting <math>1 - 0.1151</math>. Must be seen in (b) in connection with finding <math>d</math>  B1 for <math>z = \pm 1.2</math>. They must state <math>z = \pm 1.2</math> or imply it is a <math>z</math> value by its use.  This mark is only available in part (b).  2<sup>nd</sup> M1 for <math>\left(\frac{d-30}{5}\right) =</math> their negative <math>z</math> value (or equivalent)</p> <p>(c) M1 for a full method to find <math>e</math>. If they used <math>z = 1.2</math> in (b) they can get M1 for <math>z = \pm 1.2</math> here  If they use symmetry about the mean <math>\mu + (\mu - \text{their } d)</math> then ft their <math>d</math> for M1  Must explicitly <u>see</u> the method used unless the answer is correct.</p> <p>(d) M1 for a complete method or use of a correct expression e.g. “their 0.8849” - 0.1151  <u>or If their <math>d &lt;</math> their <math>e</math> using their values with <math>P(X &lt; e) - P(X &lt; d)</math></u>  If their <math>d \geq</math> their <math>e</math> then they can only score from an argument like <math>1 - 2 \times 0.1151</math>  A negative probability or probability <math>&gt; 1</math> for part (d) scores M0A0</p>	

Question Number	Scheme	Marks
Q8 (a)	<p>Let the random variable <math>X</math> be the lifetime in hours of bulb</p> $P(X < 830) = P\left(Z < \frac{\pm(830 - 850)}{50}\right)$ <p style="text-align: right;">Standardising with 850 and 50</p> $= P(Z < -0.4)$ $= 1 - P(Z < 0.4)$ <p style="text-align: right;">Using 1-(probability&gt;0.5)</p> $= 1 - 0.6554$ $= 0.3446 \text{ or } 0.344578 \text{ by calculator}$ <p style="text-align: right;">awrt 0.345</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p>
(b)	$0.3446 \times 500$ $= 172.3$ <p style="text-align: right;">Their (a) x 500</p> <p style="text-align: right;">Accept 172.3 or 172 or 173</p>	<p>M1</p> <p>A1</p> <p>(2)</p>
(c)	<p>Standardise with 860 and <math>\sigma</math> and equate to <math>z</math> value <math>\frac{\pm(818 - 860)}{\sigma} = z</math> value</p> $\frac{818 - 860}{\sigma} = -0.84(16) \text{ or } \frac{860 - 818}{\sigma} = 0.84(16) \text{ or } \frac{902 - 860}{\sigma} = 0.84(16) \text{ or equiv.}$ <p style="text-align: right;"><math>\pm 0.8416(2)</math></p> $\sigma = 49.9$ <p style="text-align: right;">50 or awrt 49.9</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>A1</p> <p>(4)</p>
(d)	<p>Company <math>Y</math> as the <u>mean</u> is greater for <math>Y</math>.</p> <p>They have (approximately) the same <u>standard deviation</u> or <u>sd</u></p> <p style="text-align: right;">both</p>	<p>B1</p> <p>B1</p> <p>(2)</p> <p>[11]</p>
Notes	<p>8(a) If 1-<math>z</math> used e.g. 1-0.4=0.6 then award second M0</p> <p>8(c) M1 can be implied by correct line 2</p> <p>A1 for completely correct statement or equivalent.</p> <p>Award B1 if 0.8416(2) seen</p> <p>Do not award final A1 if any errors in solution e.g. negative sign lost.</p> <p>8(d) Must use statistical terms as underlined.</p>	

Question Number	Scheme	Marks
Q7 (a)	 <p style="text-align: right;">bell shaped, must have inflexions 154,172 on axis 5% and 30%</p>	<p>B1 B1 B1 (3)</p>
(b)	$P(X < 154) = 0.05$ $\frac{154 - \mu}{\sigma} = -1.6449 \quad \text{or} \quad \frac{\mu - 154}{\sigma} = 1.6449$ $\mu = 154 + 1.6449\sigma \quad \text{**given**}$	<p>M1 B1 A1 cso (3)</p>
(c)	$172 - \mu = 0.5244\sigma \quad \text{or} \quad \frac{172 - \mu}{\sigma} = 0.5244$ <p style="text-align: right;">(allow <math>z = 0.52</math> or better here but must be in an equation)</p> <p>Solving gives <math>\sigma = 8.2976075</math> (<b>awrt 8.30</b>) and <math>\mu = 167.64873</math> (<b>awrt 168</b>)</p>	<p>B1 M1 A1 A1 (4)</p>
(d)	$P(\text{Taller than 160cm}) = P\left(Z > \frac{160 - \mu}{\sigma}\right)$ $= P(Z < 0.9217994)$ $= 0.8212$ <p style="text-align: right;"><b>awrt 0.82</b></p>	<p>M1 B1 A1 (3) <b>Total [13]</b></p>
(a)	<p>2<sup>nd</sup> B1 for 154 and 172 marked but 154 must be <math>&lt; \mu</math> and <math>172 &gt; \mu</math>. But <math>\mu</math> need not be marked. Allow for <math>\frac{154 - \mu}{\sigma}</math> and <math>\frac{172 - \mu}{\sigma}</math> marked on appropriate sides of the peak.</p> <p>3<sup>rd</sup> B1 the 5% and 30% should be clearly indicated in the correct regions i.e. LH tail and RH tail.</p>	
(b)	<p>M1 for <math>\pm \frac{(154 - \mu)}{\sigma} = z</math> value (<math>z</math> must be recognizable e.g. 1.64, 1.65, 1.96 but NOT 0.5199 etc)</p> <p>B1 for <math>\pm 1.6449</math> seen in a line before the final answer.</p> <p>A1cso for no incorrect statements (in <math>\mu</math>, <math>\sigma</math>) equating a <math>z</math> value and a probability or incorrect signs e.g. <math>\frac{154 - \mu}{\sigma} = 0.05</math> or <math>\frac{154 - \mu}{\sigma} = 1.6449</math> or <math>P(Z &lt; \frac{\mu - 154}{\sigma}) = 1.6449</math></p>	
(c)	<p>B1 for a correct 2<sup>nd</sup> equation (NB <math>172 - \mu = 0.525\sigma</math> is B0, since <math>z</math> is incorrect)</p> <p>M1 for solving their two linear equations leading to <math>\mu = \dots</math> or <math>\sigma = \dots</math></p> <p>1<sup>st</sup> A1 for <math>\sigma =</math> awrt 8.30, 2<sup>nd</sup> A1 for <math>\mu =</math> awrt 168 [NB the 168 can come from false working. These A marks require use of correct equation from (b), and a <math>z</math> value for “0.5244” in (c)] NB use of <math>z = 0.52</math> will typically get <math>\sigma = 8.31</math> and <math>\mu = 167.67\dots</math> and score B1M1A0A1 <u>No working</u> and both correct scores 4/4, only one correct scores 0/4 Provided the M1 is scored the A1s can be scored even with B0 (e.g. for <math>z = 0.525</math>)</p>	
(d)	<p>M1 for attempt to standardise with 160, their <math>\mu</math> and their <math>\sigma (&gt; 0)</math>. Even allow with symbols <math>\mu</math> and <math>\sigma</math>.</p> <p>B1 for <math>z =</math> awrt <math>\pm 0.92</math> <u>No working</u> and a correct answer can score 3/3 provided <math>\sigma</math> and <math>\mu</math> are correct to 2sf.</p>	

Question Number	Scheme	Marks
Q7 (a)	$P(D > 20) = P\left(Z > \frac{20-30}{8}\right)$ $= P(Z > -1.25)$ $= \underline{\mathbf{0.8944}} \qquad \qquad \qquad \underline{\mathbf{awrt 0.894}}$ <p>(b) <math>P(D &lt; Q_3) = 0.75</math> so <math>\frac{Q_3 - 30}{8} = 0.67</math></p> $Q_3 = \mathbf{awrt 35.4}$ <p>(c) <math>35.4 - 30 = 5.4</math> so <math>Q_1 = 30 - 5.4 = \mathbf{awrt 24.6}</math></p> <p>(d) <math>Q_3 - Q_1 = 10.8</math> so <math>1.5(Q_3 - Q_1) = 16.2</math> so <math>Q_1 - 16.2 = h</math> or <math>Q_3 + 16.2 = k</math>  <math>h = \underline{\mathbf{8.4 to 8.6}}</math> and <math>k = \underline{\mathbf{51.4 to 51.6}}</math> both</p> <p>(e) <math>2P(D &gt; 51.6) = 2P(Z &gt; 2.7)</math></p> $= 2[1 - 0.9965] = \mathbf{awrt 0.007}$	M1 A1 A1 (3) M1 B1 A1 (3) B1ft (1) M1 A1 (2) M1 M1 A1 (3)  <b>Total 12</b>
	<p>(a) M1 for an attempt to standardise 20 or 40 using 30 and 8.  1<sup>st</sup> A1 for <math>z = \pm 1.25</math>  2<sup>nd</sup> A1 for awrt 0.894</p> <p>(b) M1 for <math>\frac{Q_3 - 30}{8} =</math> to a <math>z</math> value  M0 for 0.7734 on RHS.  B1 for (<math>z</math> value) between 0.67~0.675 seen.  M1B0A1 for use of <math>z = 0.68</math> in correct expression with awrt 35.4</p> <p>(c) Follow through using their of quartile values.</p> <p>(d) M1 for an attempt to calculate <math>1.5(IQR)</math> and attempt to add or subtract using one of the formulae given in the question - follow through their quartiles</p> <p>(e) 1<sup>st</sup> M1 for attempting <math>2P(D &gt; \text{their } k)</math> or <math>(P(D &gt; \text{their } k) + P(D &lt; \text{their } h))</math>  2<sup>nd</sup> M1 for standardising their <math>h</math> or <math>k</math> (may have missed the 2) so allow for standardising <math>P(D &gt; 51.6)</math> or <math>P(D &lt; 8.4)</math>  Require boths Ms to award A mark.</p>	