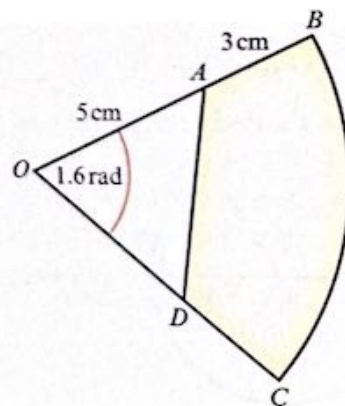


Radians

Exercise 5D

- Ⓟ 10 In the diagram, BC is the arc of a circle, centre O and radius 8 cm. The points A and D are such that $OA = OD = 5$ cm. Given that $\angle BOC = 1.6$ radians, calculate the area of the shaded region.



Shaded region = sector - triangle

$$= \frac{1}{2} r^2 \theta - \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 8^2 \times 1.6 - \frac{1}{2} \times 5 \times 5 \times \sin 1.6$$

$$= 38.7 \text{ cm}^2$$

- Ⓟ 12 In the diagram, AD and BC are arcs of circles with centre O , such that $OA = OD = r$ cm, $AB = DC = 8$ cm and $\angle BOC = \theta$ radians.

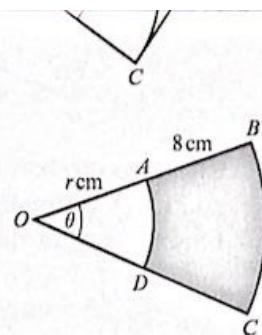
a Given that the area of the shaded region is 48 cm^2 , show that

$$r = \frac{6}{\theta} - 4$$

(4 marks)

b Given also that $r = 10\theta$, calculate the perimeter of the shaded region.

(6 marks)



a) Shaded Area = large sector - small sector

$$48 \text{ cm}^2 = \frac{1}{2} (r+8)^2 \theta - \frac{1}{2} r^2 \theta$$

$$96 = (r^2 + 16r + 64) \theta - r^2 \theta$$

$$96 = 16r\theta + 64\theta$$

$$6 = r\theta + 4\theta$$

$$6 - 4\theta = r\theta$$

$$\underline{\underline{\frac{6}{\theta} - 4 = r}}$$

b)

$$\text{Perimeter} = \text{Big arc} + \text{Small arc} + 8 + 8$$

$$= (r+8)\theta + r\theta + 16$$

$$= (10\theta + 8)\theta + 10\theta^2 + 16$$

$$= \underline{\underline{20\theta^2 + 8\theta + 16}}$$

Arc length
= $r\theta$

From part a)

$$\frac{6}{\theta} - 4 = 10\theta$$

$$6 - 4\theta = 10\theta^2$$

$$3 - 2\theta = 5\theta^2$$

$$5\theta^2 + 2\theta - 3 = 0$$

$$(5\theta - 3)(\theta + 1) = 0$$

$$\underline{\underline{\theta = \frac{3}{5}}} \quad \text{or} \quad \cancel{\theta = -1}$$

$$\text{Perimeter} = 20(0.6)^2 + 8(0.6) + 16 = 28 \text{ cm}$$

- E/P** 16 The diagram shows the sector OAB of a circle with centre O , radius 12 cm and angle 1.2 radians.

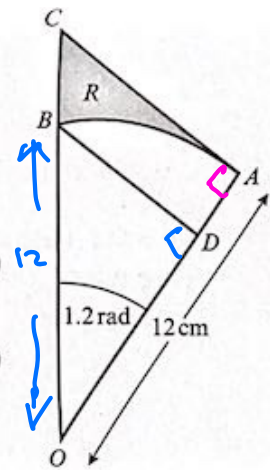
The line AC is a tangent to the circle with centre O , and OBC is a straight line.

The region R is bounded by the arc AB and the lines AC and CB .

- a Find the area of R , giving your answer to 2 decimal places. (8 marks)

The line BD is parallel to AC .

- b Find the perimeter of DAB . (5 marks)



$$a) R = \Delta OAC - \text{Sector } OAB$$

$$\tan(1.2) = \frac{AC}{12}$$

$$AC = 12 \tan 1.2$$

$$\begin{aligned} \text{Then area of } \Delta OAC &= \frac{1}{2} \times 12 \times 12 \tan 1.2 \\ &= 185.1949 \end{aligned}$$

$$\begin{aligned} \text{Area of sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 12^2 \times 1.2 = 86.4 \text{ cm}^2 \end{aligned}$$

$$R = 185.1949 - 86.4$$

$$= 98.7949$$

$$= 98.79 \text{ cm}^2 \text{ to 2 d.p.}$$

$$\text{In } \Delta ODB \quad \sin(1.2) = \frac{BD}{12}$$

$$BD = 12 \sin(1.2)$$

$$\cos(1.2) = \frac{OD}{12}$$

$$OD = 12 \cos(1.2)$$

$$\begin{aligned} AD &= 12 - OD \\ &= 12 - 12 \cos(1.2) \end{aligned}$$

$$\text{Perimeter of ABD} = \text{Arc AB} + BD + AD$$

(r θ)

$$= 12(1.2) + 12 \sin(1.2) + 12 - 12 \cos(1.2)$$

$$= 33.24 \text{ cm}$$
