Integration - Mixed Exercise 13

2) $f'(x) = x^2 - 3x - \frac{2}{x^2}$ through $(1,1)$

$f'(x) = x^2 - 3x - 2x^{-2}$

$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - \frac{2x^{-1}}{-1} + c$

$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + \frac{2}{x} + c$

$f(1) = \frac{1}{3} - \frac{3}{2} + \frac{2}{1} + c = 1$

$c = 1 - \frac{1}{3} + \frac{3}{2} - 2 = \frac{1}{6}$

$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + \frac{2}{x} + \frac{1}{6}$

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4) $y = \frac{(x+1)(2x-3)}{\sqrt{x}}$ find $\int y\,dx$

$y = \frac{2x^2 - x - 3}{\sqrt{x}}$

$y = 2x^{3/2} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$

$\int y\,dx = \frac{2x^{5/2}}{5/2} - \frac{x^{3/2}}{3/2} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$$= \frac{4}{5}x^{5/2} - \frac{2}{3}x^{3/2} - 6x^{\frac{1}{2}} + C$$

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6)

$$y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$$

a)

$$y = \left(x^{\frac{1}{3}} + 3\right)^2$$

$$= x^{2/3} + 6x^{\frac{1}{3}} + 9$$

$A = 6$
$B = 9$

b)

$$\int y\,dx = \frac{x^{5/3}}{5/3} + \frac{6x^{4/3}}{4/3} + 9x + C$$

$$= \frac{3}{5}x^{5/3} + \frac{9}{2}x^{4/3} + 9x + C$$

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8)

$$\int\left(\frac{a}{3x^3} - ab\right)dx = -\frac{2}{3x^2} + 14x + C$$

$$\int\left(\frac{ax^{-3}}{3} - ab\right)dx = -\frac{2}{3x^2} + 14x + C$$

$$\frac{ax^{-2}}{3(-2)} - abx + C = -\frac{2}{3x^2} + 14x + C$$

$$-\frac{a}{6x^2} - abx + C = -\frac{2}{3x^2} + 14x + C$$

$$-\frac{a}{6} = -\frac{2}{3} \quad \Rightarrow \quad a = 4$$

$$-ab = 14$$

$$-4b = 14$$

$$b = \frac{14}{-4} = -\frac{7}{2}$$

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10) $f'(t) = 5 + 2t \qquad f(0) = 0$

a) $f(t) = \int (5 + 2t)\,dt$

$= 5t + t^2 + c$

$f(0) = 0 + 0 + c \quad \Rightarrow \quad c = 0$

$$\underline{f(t) = 5t + t^2}$$

b) If $f(t) = 100$

$5t + t^2 = 100$

$t^2 + 5t - 100 = 0$

$\underline{t = 7.81\ s}$ or ~~$t = -12.8$~~

12) a) $\int (x^{\frac{1}{2}} - 4)(x^{-\frac{1}{2}} - 1)\,dx$

$= \int [x^0 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4]\,dx$

$= \int (5 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}})\,dx$

$= 5x - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{3/2}}{3/2} + c$

$= 5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{3/2} + c$

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b) $\int_1^4 (x^{\frac{1}{2}} - 4)(x^{-\frac{1}{2}} - 1)\,dx$

$= \left[5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{3/2}\right]_1^4$

$= \left(5(4) - 8(2) - \frac{2}{3}(8)\right) - \left(5 - 8 - \frac{2}{3}\right)$

$= 20 - 16 - \frac{16}{3} - 5 + 8 + \frac{2}{3}$

$= 7 - \frac{14}{3} = \frac{7}{3}$

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14) $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$ $\quad x > 0$

a) $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-3/2}$

b) $\int y\,dx = \int \left(3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}\right)dx$

$= \frac{3x^{3/2}}{3/2} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2x^{3/2} - 8x^{\frac{1}{2}} + c$

c) $\int_1^3 y\,dx = \left[2x^{3/2} - 8x^{\frac{1}{2}}\right]_1^3$

$= \left(2(3\sqrt{3}) - 8\sqrt{3}\right) - \left(2 - 8\right)$

$= 6\sqrt{3} - 8\sqrt{3} - 2 + 8$

$= 6 - 2\sqrt{3}$

$A = 6$
$B = -2$