

# Composite Functions

Function notation  $f(x) = 2x + 3$

$$f(3) = 2(3) + 3 = 9$$

$$f(-2) = 2(-2) + 3 = -1$$

$$f(0) = 2(0) + 3 = 3$$

$$g(x) = x^2$$

$$g(4) = 4^2 = 16$$

$$g(-4) = (-4)^2 = 16$$

Composite functions arise when the output of one function becomes the input of another.

Example  $f(x) = 2x + 3$ ,  $g(x) = x^2$

$$fg(x) = f(x^2) = 2x^2 + 3$$

$$gf(x) = g(2x + 3) = (2x + 3)^2$$

$$\begin{aligned} &\text{or } (2x + 3)(2x + 3) \\ &= 4x^2 + 6x + 6x + 9 \\ &= 4x^2 + 12x + 9 \end{aligned}$$

Notice  $fg(x) \neq gf(x)$

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Examples

$$f(x) = 4x + 1$$

$$g(x) = x^2$$

$$h(x) = \frac{4}{x}$$

Find

$$\begin{aligned} 1) \quad fg(3) &= f(3^2) \\ &= f(9) \\ &= 4(9) + 1 \\ &= 37 \end{aligned}$$

$$\begin{aligned} 2) \quad gh(2) &= g\left(\frac{4}{2}\right) \\ &= g(2) \\ &= 2^2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} 3) \quad hfg(x) &= hf(x^2) \\ &= h(4x^2 + 1) \\ &= \frac{4}{4x^2 + 1} \end{aligned}$$

Exercise

$$f(x) = 2x - 3$$

$$g(x) = x^3$$

$$h(x) = 4x$$

Find

$$1) \quad hf(5)$$

$$4) \quad hg(1)$$

$$2) \quad f(-2)$$

$$5) \quad fh(x)$$

$$3) \quad gh(x)$$

$$1) \quad hf(5) = h(2(5)-3) = h(7) = 4 \times 7 = 28$$

$$2) \quad f(-2) = 2(-2) - 3 = -4 - 3 = -7$$

$$3) \quad gh(x) = g(4x) = (4x)^3 = 64x^3$$

$$4) \quad hg(1) = h(1^3) = h(1) = 4 \times 1 = 4$$

$$5) \quad fh(x) = f(4x) = 2(4x) - 3 = 8x - 3$$

11 f and g are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

(a) Find  $f(-5)$

$$f(-5) = \frac{2}{(-5)^2} = \frac{2}{25}$$

$$\frac{2}{25}$$

(1)

(b) Find  $fg(1)$

$$\begin{aligned} fg(1) &= f(4 \times 1^3) \\ &= f(4) \\ &= \frac{2}{4^2} = \frac{2}{16} = \frac{1}{8} \end{aligned}$$

$$\frac{1}{8}$$

(2)

(Total for Question 11 is 3 marks)

Can also have  $ff(x)$  or  $f^2(x)$

$$\text{If } f(x) = 2x - 3$$

$$\begin{aligned} ff(x) &= f(2x-3) \\ &= 2(2x-3) - 3 \\ &= 4x - 6 - 3 \\ &= 4x - 9 \end{aligned}$$

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$$f(x) = 2x - 3$$

$$g(x) = x^3$$

$$h(x) = 4x$$

Find  $gg(x)$  and  $hh(x)$

$$gg(x) = g(x^3) = (x^3)^3 = x^9$$

$$hh(x) = h(4x) = 16x$$

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