Stationary Points and Modelling Questions (52 Marks)

Q1.

The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \quad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

(Total 9 marks)

Q2.

The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \quad x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\frac{d^2y}{dx^2}$.

(2)

(c) State the nature of the turning point.

(1)

(Total 10 marks)

Q3.

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

(Total 10 marks)

Q4.

A solid right circular cylinder has radius *r* cm and height *h* cm.

The total surface area of the cylinder is 800 cm^2.

(a) Show that the volume, *V* cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that *r* varies,

(b) use calculus to find the maximum value of *V*, to the nearest cm^3.

(6)

(c) Justify that the value of *V* you have found is a maximum.

(2)
(Total 12 marks)

Q5.

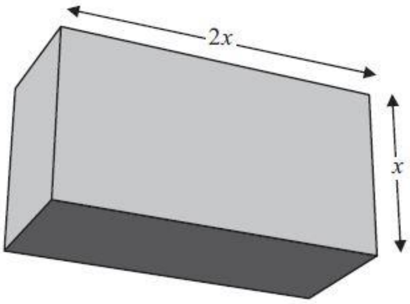

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, *x* cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, *L* cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of *L*.

(6)

(c) Justify, by further differentiation, that the value of *L* that you have found is a minimum.

(2)
(Total 11 marks)