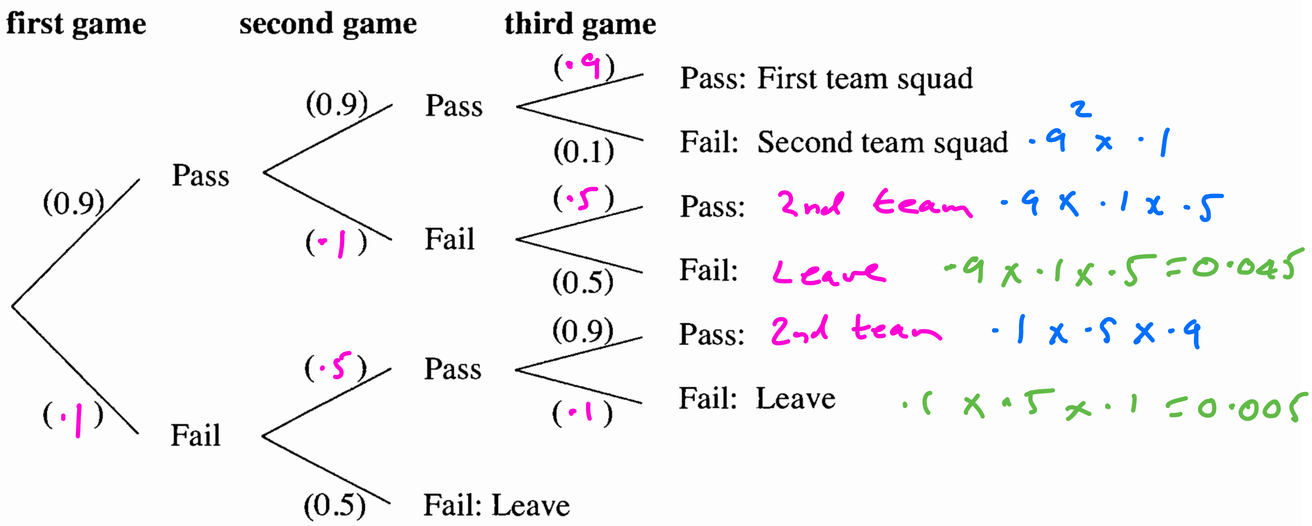
Section B (36 marks)

6 Answer part (i) of this question on the insert provided.

Mancaster Hockey Club invite prospective new players to take part in a series of three trial games. At the end of each game the performance of each player is assessed as pass or fail. Players who achieve a pass in all three games are invited to join the first team squad. Players who achieve a pass in two games are invited to join the second team squad. Players who fail in two games are asked to leave. This may happen after two games.

- The probability of passing the first game is 0.9
- Players who pass any game have probability 0.9 of passing the next game
- Players who fail any game have probability 0.5 of failing the next game

(i) On the insert, complete the tree diagram which illustrates the information above. [2]



(ii) Find the probability that a randomly selected player

(*A*) is invited to join the first team squad, $0\cdot9^3 = 0.729$ [2]

(*B*) is invited to join the second team squad. $\cdot081 + 0\cdot045 + 0\cdot045 = 0\cdot171$ [3]

(iii) Hence write down the probability that a randomly selected player is asked to leave. [1]

$1 - 0\cdot729 - 0\cdot171 = 0\cdot1$

(iv) Find the probability that a randomly selected player is asked to leave after two games, given that the player is asked to leave. [2]

Angela, Bryony and Shareen attend the trials at the same time. Assuming their performances are independent, find the probability that

(v) at least one of the three is asked to leave, [3]

(vi) they pass a total of 7 games between them. [5]

iv) $\dfrac{P(\text{Leave after 2} \cap \text{Leave})}{P(\text{Leave})} = \dfrac{0.1 \times 0.5}{0.1}$

$= 0.5$

v) P(At least one leaves)

$= 1 - P(\text{All stay})$

Prob (an individual stays) = 0.9

$= 1 - 0.9^3$

$= 1 - 0.729 \quad = \quad 0.271$

vi) 331 or 322

$P(331) = 3 \times 0.9^3 \times 0.9^3 \times (0.045 + 0.05)$

0.07971615

$P(3,2,2) = 3 \times 0.729 \times 0.171^2$

$= 0.063950$

$= 0.1437$

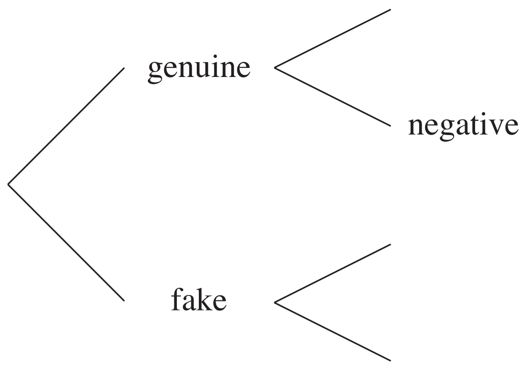
Section B (36 marks)

6 It has been estimated that 90% of paintings offered for sale at a particular auction house are genuine, and that the other 10% are fakes. The auction house has a test to determine whether or not a given painting is genuine. If this test gives a positive result, it suggests that the painting is genuine. A negative result suggests that the painting is a fake.

If a painting is genuine, the probability that the test result is positive is 0.95.

If a painting is a fake, the probability that the test result is positive is 0.2.

(i) Copy and complete the probability tree diagram below, to illustrate the information above. [2]



Calculate the probabilities of the following events.

(ii) The test gives a positive result. [2]

(iii) The test gives a correct result. [2]

(iv) The painting is genuine, given a positive result. [3]

(v) The painting is a fake, given a negative result. [3]

A second test is more accurate, but very expensive. The auction house has a policy of only using this second test on those paintings with a negative result on the original test.

(vi) Using your answers to parts **(iv)** and **(v)**, explain why the auction house has this policy. [2]

The probability that the second test gives a correct result is 0.96 whether the painting is genuine or a fake.

(vii) Three paintings are independently offered for sale at the auction house. Calculate the probability that all three paintings are genuine, are judged to be fakes in the first test, but are judged to be genuine in the second test. [4]

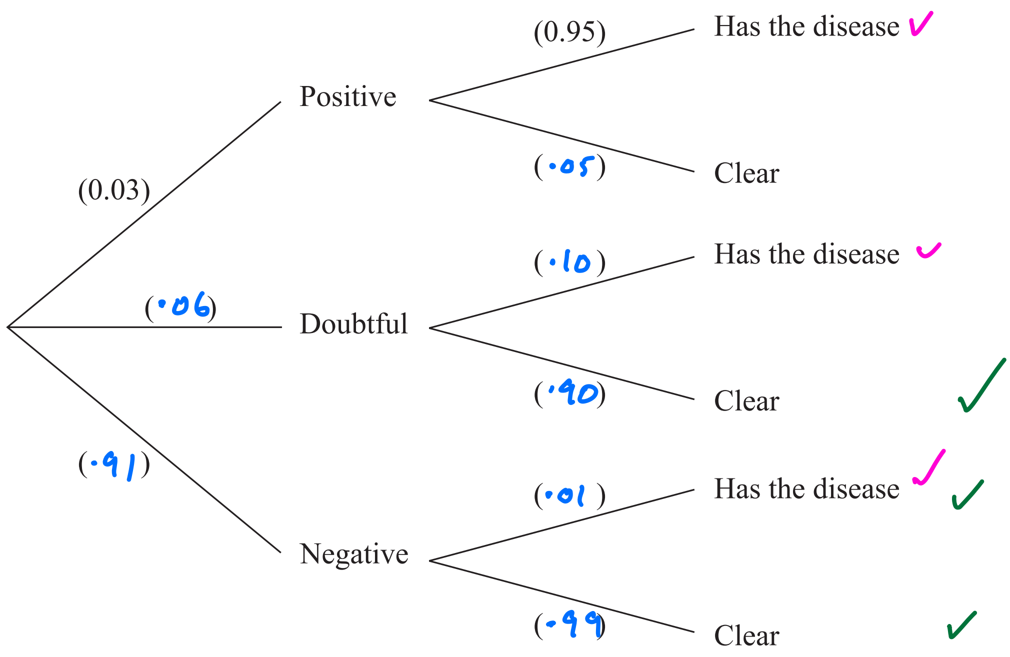
Section B (36 marks)

7 A screening test for a particular disease is applied to everyone in a large population. The test classifies people into three groups: 'positive', 'doubtful' and 'negative'. Of the population, 3% is classified as positive, 6% as doubtful and the rest negative.

In fact, of the people who test positive, only 95% have the disease. Of the people who test doubtful, 10% have the disease. Of the people who test negative, 1% actually have the disease.

People who do not have the disease are described as 'clear'.

(i) Copy and complete the tree diagram to show this information. [4]



(ii) Find the probability that a randomly selected person tests negative and is clear. [2]

(iii) Find the probability that a randomly selected person has the disease. [3]

(iv) Find the probability that a randomly selected person tests negative **given** that the person has the disease. [3]

(v) Comment briefly on what your answer to part **(iv)** indicates about the effectiveness of the screening test. [2]

Once the test has been carried out, those people who test doubtful are given a detailed medical examination. If a person has the disease the examination will correctly identify this in 98% of cases. If a person is clear, the examination will always correctly identify this.

(vi) A person is selected at random. Find the probability that this person either tests negative originally or tests doubtful and is then cleared in the detailed medical examination. [4]

ii) $P(\text{Neg} \cap \text{Clear}) = 0.91 \times 0.99 = 0.9009$

iii) $P(\text{Has Disease}) = 0.03 \times 0.95 + 0.06 \times 0.1 + 0.91 \times .01$

$= 0.0436$

iv) $$P(\text{Neg} \mid \text{Disease}) = \frac{P(\text{Neg} \cap \text{Disease})}{P(\text{Has Disease})}$$

$$= \frac{0.91 \times 0.01}{0.0436}$$

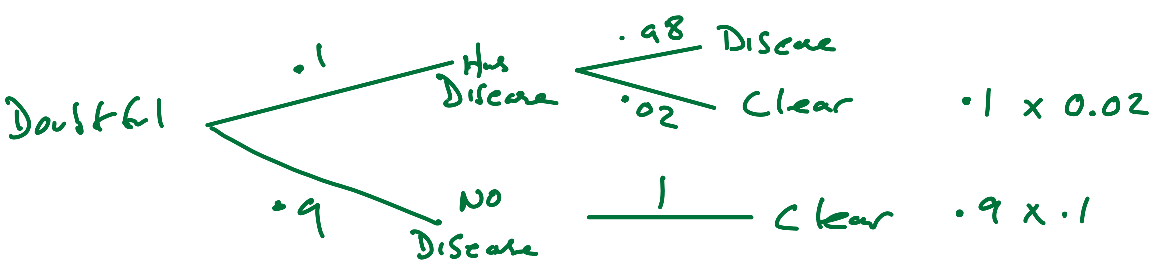
$$= 0.2087$$

v) This test fails to pick up almost 21% of people with the disease, so perhaps would be judged as unsatisfactory on its own

vi)

Once the test has been carried out, those people who test doubtful are given a detailed medical examination. If a person has the disease the examination will correctly identify this in 98% of cases. If a person is clear, the examination will always correctly identify this.

(vi) A person is selected at random. Find the probability that this person either tests negative originally or tests doubtful and is then cleared in the detailed medical examination. [4]



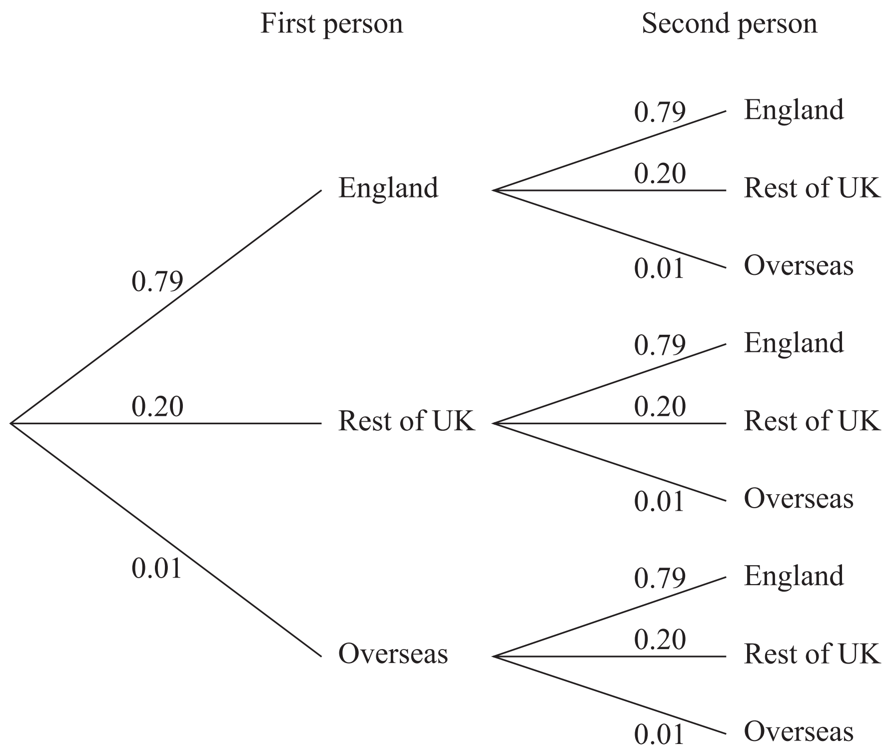
$0·91 \times 0·06 \times (·1 \times ·02 + 0.9 \times 1)$

$= 0.96412$

Section B (36 marks)

6 In a large town, 79% of the population were born in England, 20% in the rest of the UK and the remaining 1% overseas. Two people are selected at random.

You may use the tree diagram below in answering this question.



(i) Find the probability that

(A) both of these people were born in the rest of the UK, [2]

(B) at least one of these people was born in England, [3]

(C) neither of these people was born overseas. [2]

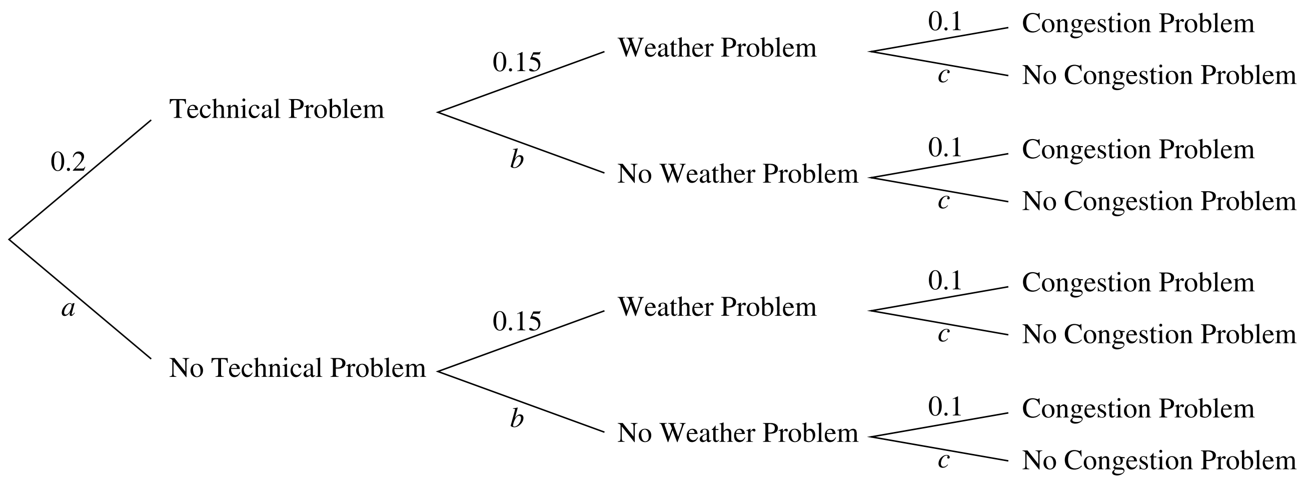
(ii) Find the probability that both of these people were born in the rest of the UK given that neither was born overseas. [3]

(iii) (A) Five people are selected at random. Find the probability that at least one of them was not born in England. [3]

(B) An interviewer selects n people at random. The interviewer wishes to ensure that the probability that at least one of them was not born in England is more than 90%. Find the least possible value of n. You must show working to justify your answer. [3]

Section B (36 marks)

7 Laura frequently flies to business meetings and often finds that her flights are delayed. A flight may be delayed due to technical problems, weather problems or congestion problems, with probabilities 0.2, 0.15 and 0.1 respectively. The tree diagram shows this information.



(i) Write down the values of the probabilities a, b and c shown in the tree diagram. **[2]**

One of Laura's flights is selected at random.

(ii) Find the probability that Laura's flight is not delayed and hence write down the probability that it is delayed. **[4]**

(iii) Find the probability that Laura's flight is delayed due to just one of the three problems. **[4]**

(iv) Given that Laura's flight is delayed, find the probability that the delay is due to just one of the three problems. **[3]**

(v) Given that Laura's flight has no technical problems, find the probability that it is delayed. **[3]**

(vi) In a particular year, Laura has 110 flights. Find the expected number of flights that are delayed. **[2]**

[redacted]

7 One train leaves a station each hour. The train is either on time or late. If the train is on time, the probability that the next train is on time is 0.95. If the train is late, the probability that the next train is on time is 0.6. On a particular day, the 09 00 train is on time.

(i) Illustrate the possible outcomes for the 10 00, 11 00 and 12 00 trains on a probability tree diagram. **[4]**

(ii) Find the probability that

(A) all three of these trains are on time, **[2]**

(B) just one of these three trains is on time, **[4]**

(C) the 12 00 train is on time. **[4]**

(iii) Given that the 12 00 train is on time, find the probability that the 10 00 train is also on time. **[4]**