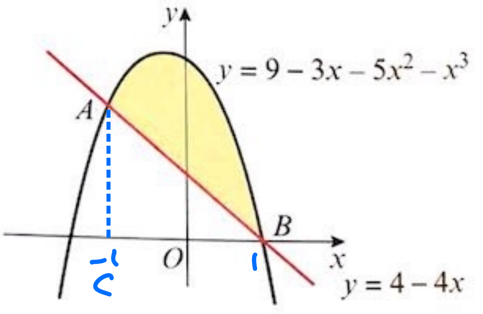
Ⓟ **3** The diagram shows a sketch of part of the curve with equation $y = 9 - 3x - 5x^2 - x^3$ and the line with equation $y = 4 - 4x$.
The line cuts the curve at the points $A\ (-1, 8)$ and $B(1, 0)$.
Find the area of the shaded region between AB and the curve.



Method 1

Required Area $= \int_{-1}^{1} (9 - 3x - 5x^2 - x^3)\,dx$ − Area of $\triangle ACB$

$= \left[9x - \frac{3x^2}{2} - \frac{5x^3}{3} - \frac{x^4}{4} \right]_{-1}^{1}$ − $\frac{1}{2}$ base × height

$= \left(9 - \frac{3}{2} - \frac{5}{3} - \frac{1}{4}\right) - \left(-9 - \frac{3}{2} + \frac{5}{3} - \frac{1}{4}\right)$ − $\frac{1}{2} \times 2 \times 8$

$= 18 - \frac{10}{3}$ − 8

$= \frac{20}{3}$ or $6\frac{2}{3}$ units^2

Method 2 Area between 2 curves

Area $= \int_{-1}^{1} \left((9 - 3x - 5x^2 - x^3) - (4 - 4x) \right) dx$

$= \int_{-1}^{1} (5 + x - 5x^2 - x^3)\,dx$

$$= \left[5x + \frac{x^2}{2} - \frac{5x^3}{3} - \frac{x^4}{4}\right]_{-1}^{1}$$

$$= \left(5 + \frac{1}{2} - \frac{5}{3} - \frac{1}{4}\right) - \left(-5 + \frac{1}{2} + \frac{5}{3} - \frac{1}{4}\right)$$

$$= 10 - \frac{10}{3} \quad = \frac{20}{3} \quad = 6\frac{2}{3}$$

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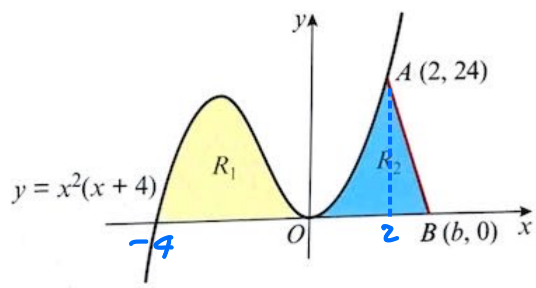
10 The sketch shows part of the curve with equation $y = x^2(x+4)$. The finite region R_1 is bounded by the curve and the negative x-axis. The finite region R_2 is bounded by the curve, the positive x-axis and AB, where $A(2, 24)$ and $B(b, 0)$.

The area of R_1 = the area of R_2.

a Find the area of R_1.

b Find the value of b.



Problem-solving

Split R_2 into two areas by drawing a vertical line at $x = 2$.

a) $$R_1 = \int_{-4}^{0} x^2(x+4)\,dx$$

$$R_1 = \int_{-4}^{0} (x^3 + 4x^2)\,dx$$

$$= \left[\frac{x^4}{4} + \frac{4x^3}{3}\right]_{-4}^{0}$$

$$= (0 + 0) - \left(64 - \frac{256}{3}\right) = \frac{64}{3} \text{ units}^2$$

b)

$$R_2 = \int_0^2 (x^3 + 4x^2)\,dx + \text{Area of } \triangle$$

$$= \left[\frac{x^4}{4} + \frac{4x^3}{3}\right]_0^2 + \frac{1}{2}(b-2) \times 24$$

$$= \left(\frac{2^4}{4} + \frac{4(2)^3}{3}\right) - (0+0) + 12(b-2)$$

$$= 4 + \frac{32}{3} + 12b - 24$$

$$= 12b - \frac{28}{3}$$

Given $R_1 = R_2$

$$\frac{64}{3} = 12b - \frac{28}{3}$$

$$\frac{64}{3} + \frac{28}{3} = 12b$$

$$\frac{92}{3} = 12b$$

$$\frac{92}{36} = b$$

$$b = \frac{23}{9} = 2\frac{5}{9}$$

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