$$x^3 - 5x - 3 = 0$$

Show this has a solution between $x = 2$ and $x = 3$

$x = 2$ $\quad 2^3 - 5(2) - 3 = 8 - 10 - 3 = -5$

$x = 3$ $\quad 3^3 - 5(3) - 3 = 27 - 15 - 3 = +9$

sign change on continuous function so $x^3 - 5x - 3 = 0$ has solution between $x = 2$ and $x = 3$ (2)

Show $x^3 - 5x - 3 = 0$ can be rearranged to give $x = \sqrt[3]{5x+3}$

$$x^3 - 5x - 3 = 0$$

$$x^3 = 5x + 3$$

$$x = \sqrt[3]{5x+3} \quad (1)$$

Starting with $x_0 = 2$

use 3 iterations of $x_{n+1} = \sqrt[3]{5x_n + 3}$ to estimate the solution

$$x_1 = \sqrt[3]{5(2)+3} = 2.351$$

$$x_2 = \sqrt[3]{5(2.351)+3} = 2.453$$

$$x_3 = \sqrt[3]{5(2.453)+3} = 2.481$$

$$x = 2.481$$

Ex 2 $x^3 - x^2 - 1 = 0$

Show this eqn has a root between $x = 1$ and $x = 2$

$x = 1$ $\quad 1^3 - 1^2 - 1 = -1$

$x = 2$ $\quad 2^3 - 2^2 - 1 = +3$

sign change for continous function so root between $x = 1$ and $x = 2$

Use $x_{n+1} = \sqrt[3]{x_n^2 + 1}$ 4 times

to estimate the root Start with $x_0 = 1$

$$x_1 = \sqrt[3]{1^2 + 1} = 1.2599$$

$$x_2 = \sqrt[3]{1.2599^2 + 1} = 1.3728$$

$$x_3 = \sqrt[3]{1.3728^2 + 1} = 1.4235$$

$$x_4 = \sqrt[3]{1.4235^2 + 1} = 1.4465$$

$$x = 1.4465$$
