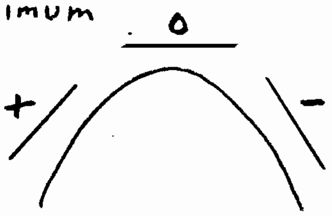
DIFFERENTIATION OF POLYNOMIAL FUNCTIONS (2) — TRANSCRIPT

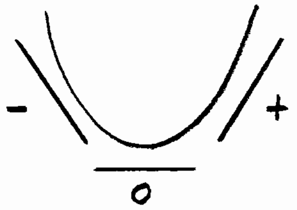
STATIONARY POINTS

Turning Points

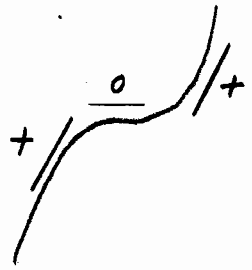
Maximum

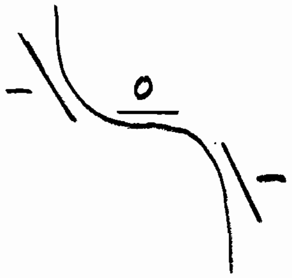


Minimum

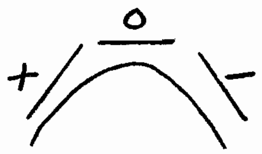


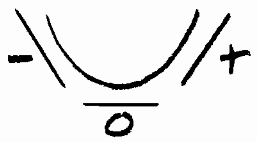
Points of Inflection





Second derivative at a stationary point: $\dfrac{d^2y}{dx^2}$

$\dfrac{d^2y}{dx^2} < 0 \Rightarrow$  Maximum

$\dfrac{d^2y}{dx^2} > 0 \Rightarrow$  Minimum

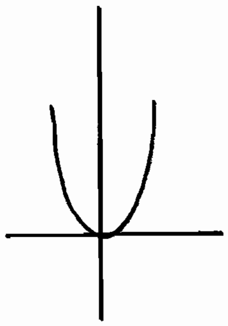
$\dfrac{d^2y}{dx^2} = 0 \Rightarrow$ not useful ?

(see next page)

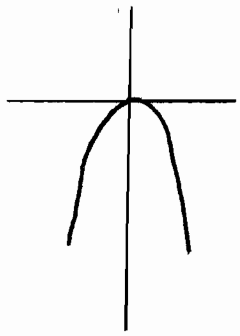
DIFFERENTIATION OF POLYNOMIAL FUNCTIONS (2) TRANSCRIPT

Consider

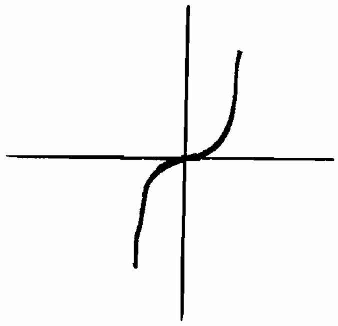
$y = x^4$	$y = -x^4$	$y = x^3$
$\frac{dy}{dx} = 4x^3$	$\frac{dy}{dx} = -4x^3$	$\frac{dy}{dx} = 3x^2$
$\frac{d^2y}{dx^2} = 12x^2$	$\frac{d^2y}{dx^2} = -12x^2$	$\frac{d^2y}{dx^2} = 6x$
At $x=0$	At $x=0$	At $x=0$
$\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 0$
$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = 0$	$\frac{d^2y}{dx^2} = 0$



A minimum



A maximum



A point of inflection

These examples show that when $\frac{d^2y}{dx^2} = 0$ at a stationary point, there may be a minimum, maximum or a point of inflection. In such cases where $\frac{d^2y}{dx^2} = 0$ it is necessary to use the alternative method for determining the nature of stationary points. That is the method where $\frac{dy}{dx}$ is calculated just before and just after the stationary point.

Example 2

Find the stationary point of the curve: $y = 2x^2 - 12x$

Indicate the nature of the stationary point and sketch the graph.

$$y = 2x^2 - 12x$$

$$\Rightarrow \frac{dy}{dx} = 4x - 12$$

At a stationary point, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 4x - 12 = 0$$

$$4x = 12$$

$$x = 3$$

When $x = 3$,

$$y = 2(3)^2 - 12(3)$$

$$= 18 - 36$$

$$= -18$$

$\therefore$ stationary point at $(3, -18)$

Checking $\frac{dy}{dx}$ just before and just after the stationary point:

When $x = 2.9$, $\frac{dy}{dx} = 4(2.9) - 12$

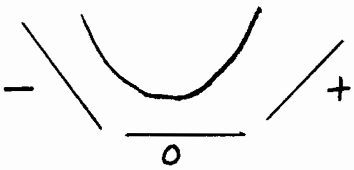
$$= 11.6 - 12$$

$$= -0.4 \quad < 0 \quad -ve$$

When $x = 3.1$, $\frac{dy}{dx} = 4(3.1) - 12$

$$= 12.4 - 12$$

$$= 0.4 > 0 \qquad +ve$$



$\therefore$ a minimum at $(3, -18)$

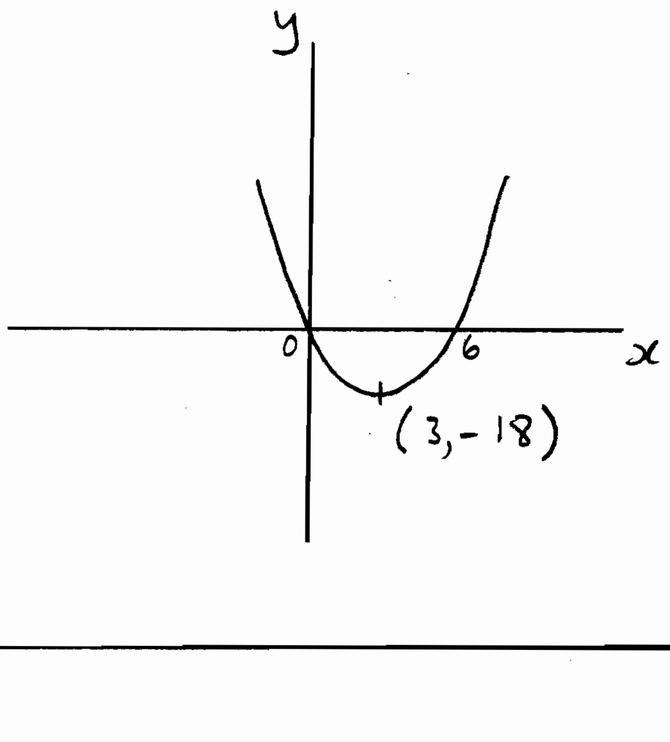
Using second derivative method

$$\frac{dy}{dx} = 4x - 12$$

$$\Rightarrow \frac{d^2y}{dx^2} = 4$$ (which does not depend on x)

If 2nd derivative is +ve at a stationary point, then the stationary point is a minimum

$\therefore$ a minimum at $(3, -18)$



Example 3

Find the turning points of the curve: $y = 2x^3 - 3x^2 - 12x + 10$

Indicate the nature of the turning points and sketch the graph.

$$y = 2x^3 - 3x^2 - 12x + 10$$

$$\Rightarrow \quad \frac{dy}{dx} = 6x^2 - 6x - 12$$

At a stationary point $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 6x^2 - 6x - 12 = 0$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad (x - 2)(x + 1) = 0$$

$$\Rightarrow \quad x = 2 \text{ or } x = -1$$

When $x = -1$, $y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 10$

$= -2 - 3 + 12 + 10$

$= 17$

Stationary point at $(-1, 17)$

When $x = 2$, $y = 2(2)^3 - 3(2)^2 - 12(2) + 10$

$= 16 - 12 - 24 + 10$

$= -10$

Stationary point at $(2, -10)$

Determine nature of stationary points

$$\frac{dy}{dx} = 6x^2 - 6x - 12$$

$$\Rightarrow \frac{d^2y}{dx^2} = 12x - 6$$

When $x = -1$, $\frac{d^2y}{dx^2} = 12(-1) - 6 = -18 < 0$

$\therefore$ a maximum at $(-1, 17)$

When $x = 2$, $\frac{d^2y}{dx^2} = 12(2) - 6 = +18 > 0$

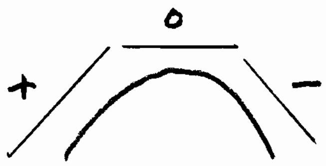
$\therefore$ a minimum at $(2, -10)$

Alternative method

Examine $\frac{dy}{dx}$ either side of $x = -1$

When $x = -1.1$, $\frac{dy}{dx} = 6(-1.1)^2 - 6(1.1) - 12 = 1.86 > 0$

when $x = -0.9$, $\frac{dy}{dx} = 6(-0.9)^2 - 6(-0.9) - 12 = -1.74 < 0$



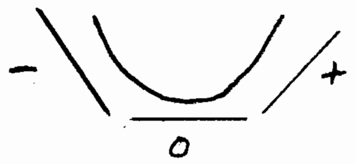
$\therefore$ a max at $(-1, 17)$

Now examine $\frac{dy}{dx}$ either side of $x = 2$

When $x = 1.9$, $\frac{dy}{dx} = 6(1.9)^2 - 6(1.9) - 12 = -1.74 < 0$

When $x = 2.1$, $\frac{dy}{dx} = 6(2.1)^2 - 6(2.1) - 12 = 1.86 > 0$

DIFFERENTIATION OF POLYNOMIAL FUNCTIONS (2) TRANSCRIPT



$\therefore$ a min at $(2, -10)$

Sketch of graph