Jan 02

6. A smooth sphere P of mass m is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass $2m$ is at rest on the table. The sphere P collides directly with Q. After the collision the direction of motion of P is unchanged. The spheres have the same radii and the coefficient of restitution between P and Q is e. By modelling the spheres as particles,

(a) show that the speed of Q immediately after the collision is $\frac{1}{3}(1 + e)u$,

(5)

(b) find the range of possible values of e.

(4)

Given that $e = \frac{1}{4}$,

(c) find the loss of kinetic energy in the collision.

(4)

(d) Give one possible form of energy into which the lost kinetic energy has been transformed.

(1)

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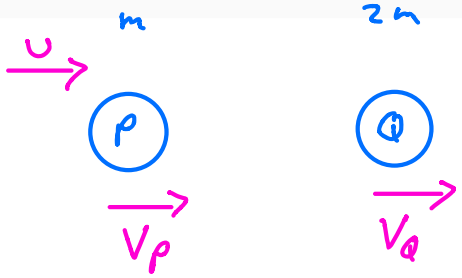
a)



PCLM $\quad mu = mV_P + 2mV_Q$

$$u = V_P + 2V_Q \qquad (1)$$

Restitution $\quad e = \dfrac{V_Q - V_P}{u}$

$$eu = V_Q - V_P \qquad (2)$$

(1) + (2) $\quad u + eu = 3V_Q$

$$\frac{U(1+e)}{3} = V_Q$$

$$V_Q = \frac{1}{3}U(1+e)$$

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b)

$$V_P = V_Q - eU$$

$$V_P = \frac{1}{3}U + \frac{1}{3}Ue - Ue$$

$$V_P = \frac{1}{3}U - \frac{2}{3}Ue$$

$$V_P = \frac{1}{3}U(1-2e)$$

Told $V_P > 0 \Rightarrow 1-2e > 0$

$$1 > 2e$$

$$\frac{1}{2} > e$$

$$0 \leq e < \frac{1}{2}$$

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c) $e = \frac{1}{4}$

$$V_P = \frac{1}{3}U\left(1-\frac{1}{4}\right) = \frac{1}{6}U$$

$$V_Q = \frac{1}{3}U\left(1+\frac{1}{4}\right) = \frac{5U}{12}$$

Initial KE $= \frac{1}{2}mU^2$

Final KE $= \frac{1}{2}m\left(\frac{U}{6}\right)^2 + \frac{1}{2}(2m)\left(\frac{5U}{12}\right)^2$

$$= \frac{mv^2}{72} + \frac{25}{144}mv^2$$

$$= \frac{3mv^2}{16}$$

Loss in KE $= \frac{1}{2}mv^2 - \frac{3}{16}mv^2$

$$= \frac{5mv^2}{16}$$

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d) Sound energy

Jan 03

6. A smooth sphere P of mass $2m$ is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass m is at rest on the table. The sphere P collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{3}$. The spheres are modelled as particles.

(a) Show that, immediately after the collision, the speeds of P and Q are $\frac{5}{9}u$ and $\frac{8}{9}u$ respectively.

(7)

After the collision, Q strikes a fixed vertical wall which is perpendicular to the direction of motion of P and Q. The coefficient of restitution between Q and the wall is e. When P and Q collide again, P is brought to rest.

(b) Find the value of e.

(7)

(c) Explain why there must be a third collision between P and Q.

(1)

Jun 01

6. A particle A of mass $2m$ is moving with speed $2u$ on a smooth horizontal table. The particle collides directly with a particle B of mass $4m$ moving with speed u in the same direction as A. The coefficient of restitution between A and B is $\frac{1}{2}$.

(a) Show that the speed of B after the collision is $\frac{3}{2}u$.

(6)

(b) Find the speed of A after the collision.

(2)

Subsequently B collides directly with a particle C of mass m which is at rest on the table. The coefficient of restitution between B and C is e. Given that there are no further collisions,

(c) find the range of possible values for e.

(8)

Jun 02

6. A small smooth ball A of mass m is moving on a horizontal table with speed u when it collides directly with another small smooth ball B of mass $3m$ which is at rest on the table. The balls have the same radius and the coefficient of restitution between the balls is e. The direction of motion of A is reversed as a result of the collision.

(a) Find, in terms of e and u. the speeds of A and B immediately after the collision.

(7)

In the subsequent motion B strikes a vertical wall, which is perpendicular to the direction of motion of B, and rebounds. The coefficient of restitution between B and the wall is $\frac{3}{4}$.

Given that there is a second collision between A and B,

(b) find the range of values of e for which the motion described is possible.

(6)

Jun 03

7. A uniform sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another uniform sphere B of mass $2m$ which is at rest on the table. The spheres are of equal radius and the coefficient of restitution between them is e. The direction of motion of A is unchanged by the collision.

(a) Find the speeds of A and B immediately after the collision.

(7)

(b) Find the range of possible values of e.

(2)

After being struck by A, the sphere B collides directly with another sphere C, of mass $4m$ and of the same size as B. The sphere C is at rest on the table immediately before being struck by B. The coefficient of restitution between B and C is also e.

(c) Show that, after B has struck C, there will be a further collision between A and B.

(6)

Hwk The questions above