Proof by Induction

Prove $\displaystyle\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

when $n=1$ $\displaystyle\sum_{r=1}^{1} r^2 = 1^2 = 1$

$\frac{1}{6}(1)(1+1)(2(1)+1) = \frac{1}{6} \times 1 \times 2 \times 3 = 1$ ✓

Formula true for $n=1$

Assume true for $n=k$

then $\displaystyle\sum_{r=1}^{k} = \frac{1}{6}k(k+1)(2k+1)$

$\frac{1}{6}(k+1)(k+2)(2k+3)$

Now consider $\displaystyle\sum_{r=1}^{k+1} r^2$

$$= \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$$

$$= \frac{1}{6}(k+1)\left[k(2k+1) + 6(k+1)\right]$$

$$= \frac{1}{6}(k+1)\left[2k^2 + k + 6k + 6\right]$$

$$= \frac{1}{6}(k+1)(k+2)(2k+3)$$

$$= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$$

This is the same formula with k replaced by $k+1$

$\therefore$ if formula true for $n=k$ it is is also true for $n=k+1$

Since formula true for $n=1$, by mathematical induction it true for all positive integers n

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Ex 2 Prove $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

When $n=1$ $1^3 = 1$ $\frac{1}{4}(1)^2(1+1)^2 = \frac{1}{4}\times 1\times 4 = 1$ ✓

$\therefore$ true for $n=1$

Assume true for $n=k$

$$\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$$

Consider $n=k+1$

$$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$$

$$= \frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right]$$

$$= \frac{1}{4}(k+1)^2\left[k^2+4k+4\right]$$

$$= \frac{1}{4}(k+1)^2(k+2)^2$$

$$= \frac{1}{4}(k+1)^2((k+1)+1)^2$$

This is same formula with k replaced by $k+1$

$\therefore$ if true for $n=k$, also true for $n=k+1$

Since true for $n=1$, by mathematical induction it is true for all positive integers n

6 Prove by induction that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. [7]

$n=1$ $\quad \frac{1}{1(1+1)} = \frac{1}{2} \qquad \frac{1}{1+1} = \frac{1}{2}$ ✓

True for $n=1$

Assume true for $n=k$

$$\sum_{r=1}^{k} \frac{1}{r(r+1)} = \frac{k}{k+1}$$

Consider $\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} = \frac{k+1}{(k+1)+1}$$

Same formula with k replaced by $k+1$

$\therefore$ if true for $n=k$ also true for $n=k+1$

Since true for $n=1$, by mathematical induction true for all positive integers n

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Hwk Exercise 8A Q1, Q3, Q5
