

Completing the Square

$$\begin{aligned}\text{Consider } (x+a)(x+a) \\&= x^2 + ax + ax + a^2 \\&= x^2 + 2ax + a^2\end{aligned}$$

Given a quadratic expression such as

$$x^2 + 6x + 10$$

we wish to write it in the form

$$(x+a)^2 + b$$

where a, b are constants

$$= (x+3)^2 + 10 - 9$$

$$= (x+3)^2 + 1$$

$$\text{but } (x+3)^2$$

$$= x^2 + 6x + 9$$

The number in the bracket $(+3)$ is half of the number of x in the expression $(+6)$

However, this introduces an extra term $(+3)^2$ which we then need to subtract to ensure the two expressions are equal

Example:

$$\begin{aligned}1) \quad x^2 + 4x - 3 &= (x+2)^2 - 3 - 4 \\&= (x+2)^2 - 7\end{aligned}$$

$$\begin{aligned}
 2) \quad x^2 - 10x + 11 &= (x-5)^2 + 11 - 25 \\
 &= (x-5)^2 - 14
 \end{aligned}$$

$$\begin{aligned}
 3) \quad x^2 + 5x + 8 &= \left(x + \frac{5}{2}\right)^2 + 8 - \frac{25}{4} \\
 &= \left(x + \frac{5}{2}\right)^2 + \frac{32}{4} - \frac{25}{4} \\
 &= \left(x + \frac{5}{2}\right)^2 + \frac{7}{4}
 \end{aligned}$$

Exercise Complete the square

$$\begin{aligned}
 1) \quad x^2 + 2x + 7 &= (x+1)^2 + 7 - 1 \\
 &= (x+1)^2 + 6
 \end{aligned}$$

$$\begin{aligned}
 2) \quad x^2 - 8x - 1 &= (x-4)^2 - 1 - 16 \\
 &= (x-4)^2 - 17
 \end{aligned}$$

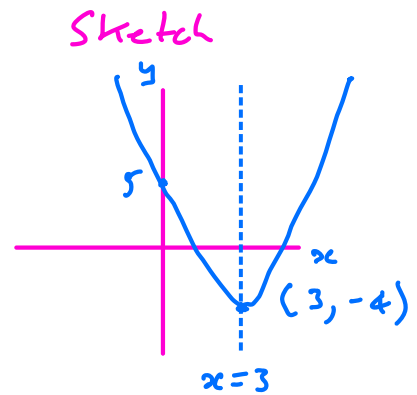
$$\begin{aligned}
 3) \quad x^2 + 3x + 5 &= \left(x + \frac{3}{2}\right)^2 + 5 - \frac{9}{4} \\
 &= \left(x + \frac{3}{2}\right)^2 + \frac{20}{4} - \frac{9}{4} \\
 &= \left(x + \frac{3}{2}\right)^2 + \frac{11}{4}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad x^2 - 9x + 10 &= \left(x - \frac{9}{2}\right)^2 + 10 - \frac{81}{4} \\
 &= \left(x - \frac{9}{2}\right)^2 + \frac{40}{4} - \frac{81}{4} \\
 &= \left(x - \frac{9}{2}\right)^2 - \frac{41}{4}
 \end{aligned}$$

Identifying Line of Symmetry and Minimum Point

Ex1

$$y = x^2 - 6x + 5$$
$$y = (x - 3)^2 + 5 - 9$$
$$y = (x - 3)^2 - 4$$



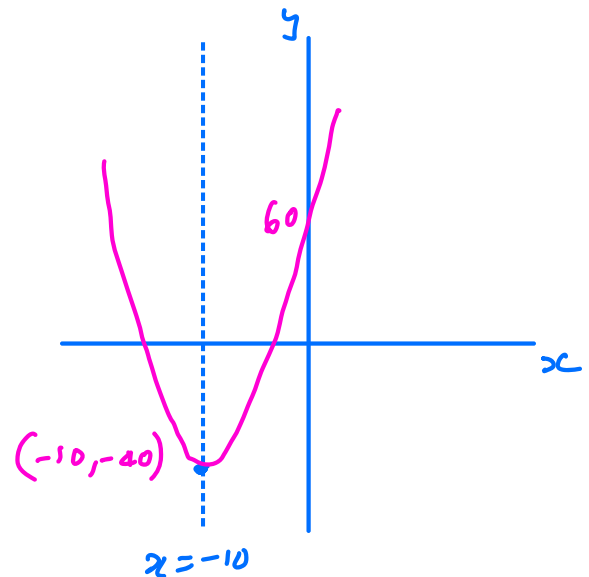
Line of symmetry $x = 3$ (value that makes bracket 0)

Minimum Point is $(3, -4)$

Notice the constant term (-4) indicates the y -coordinate of the minimum point

Ex2

$$y = x^2 + 20x + 60$$
$$y = (x + 10)^2 + 60 - 100$$
$$y = (x + 10)^2 - 40$$

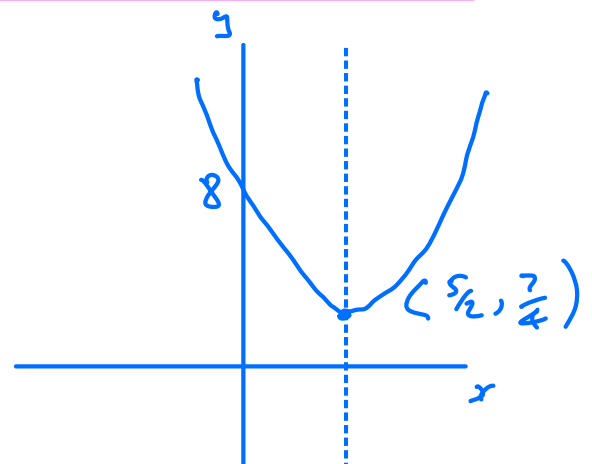


Line of symmetry $x = -10$

Min point $(-10, -40)$

Ex3

$$y = x^2 - 5x + 8$$
$$y = (x - \frac{5}{2})^2 + 8 - \frac{25}{4}$$
$$y = (x - \frac{5}{2})^2 + \frac{32}{4} - \frac{25}{4}$$
$$y = (x - \frac{5}{2})^2 + \frac{7}{4}$$



Line of symmetry = $x = \frac{5}{2}$

Min point $(\frac{5}{2}, \frac{7}{4})$

$$x = \frac{5}{2}$$

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