Matrices - Enlargements

Examples

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ by \end{pmatrix}$$

stretch by s.f a parallel to x-axis
stretch by s.f b parallel to y-axis

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ ay \end{pmatrix}$$

Enlargement by scale factor a

area scale factor $= a^2$

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1 Find the invariant points for the transformations with the following matrices.

(i) $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$

(iv) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ **(v)** $\begin{pmatrix} 4 & 1 \\ 6 & 3 \end{pmatrix}$ **(vi)** $\begin{pmatrix} 7 & -4 \\ 3 & -1 \end{pmatrix}$

1 i) $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$0x - y = x \Rightarrow x = -y$

$x + 2y = y \Rightarrow x = -y$

$y = -x$ is line of invariant points

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1 iv) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y = x \Rightarrow x + y = \sqrt{2}x$

$y = (\sqrt{2}-1)x$ (1)

$-\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y = y$

$\Rightarrow -x + y = \sqrt{2}y$

$(1-\sqrt{2})y = x$ (2)

sub for x in (1)

$y = (\sqrt{2}-1)(1-\sqrt{2})y$

$y = (\sqrt{2}-1-2+\sqrt{2})y$

$y = (2\sqrt{2}-3)y$

$\Rightarrow y = 0$

$\Rightarrow x = 0$

origin is only invariant point

Find eqns of invariant lines

Find invariant line for transformation $\begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix}$

$$\begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} X \\ mX+c \end{pmatrix}$$

$$3x - 5(mx+c) = X \quad (1)$$

$$-4x + 2(mx+c) = mX + c \quad (2)$$

From (1) $\quad 3x - 5mx - 5c = X$

$$(3-5m)x - 5c = X$$

From (2) $\quad (-4+2m)x + 2c = mX + c$

sub for X

$$(-4+2m)x + 2c = m((3-5m)x - 5c) + c$$

$$(-4+2m)x + 2c = (3m - 5m^2)x - 5mc + c$$

$$(5m^2 - m - 4)x = (-5m-2)c$$

$$(5m+4)(m-1)x = (-5m-1)c$$

$$m = -\frac{4}{5}, \quad m = 1 \qquad c = 0$$

$$\underline{y = -\frac{4}{5}x} \qquad \underline{y = x}$$

Hwk 1 hour on Exercise 7C