Leave blank

2. The probability of a bolt being faulty is 0.3. Find the probability that in a random sample of 20 bolts there are

(a) exactly 2 faulty bolts,
(2)

(b) more than 3 faulty bolts.
(2)

These bolts are sold in bags of 20. John buys 10 bags.

(c) Find the probability that exactly 6 of these bags contain more than 3 faulty bolts.
(3)

a) $X \sim B(\overset{n}{20}, \overset{p}{0.3})$

$P(X=2) = \underline{0.0278}$

b) $P(X>3) = 1 - P(X \leq 3)$

$= 1 - 0.1071 = \underline{0.8929}$

c) $Y \sim B(10, 0.8929)$

$P(Y=6) = \underline{0.0140}$

Leave blank

5. Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly.

(7)

$X \sim B(\overset{n}{40}, \overset{p}{0.3})$

$H_0 : p = 0.3$

$H_1 : p > 0.3$

where p is probability a ripe tomato has diameter greater than 4 cm

$E(X) = np = 12$

$P(X \geq 18) = 1 - P(X \leq 17)$

$= 1 - 0.9680$

$= 0.0320 < 5\%$

Reject H_0 and accept H_1

There is sufficient evidence to support the view that the new fertilizer has increased the probability a ripe tomato will have a diameter greater than 4 cm.

Leave blank

6. The probability that a sunflower plant grows over 1.5 metres high is 0.25. A random sample of 40 sunflower plants is taken and each sunflower plant is measured and its height recorded.

(a) Find the probability that the number of sunflower plants over 1.5 m high is between 8 and 13 (inclusive) using

(i) a Poisson approximation, use Binomial instead of Poisson

(ii) a Normal approximation.

(10)

(b) Write down which of the approximations used in part (a) is the most accurate estimate of the probability. You must give a reason for your answer. Omit part b

(2)

a) $X \sim B(\overset{n}{40}, \overset{p}{0.25})$

i) $P(8 \leq X \leq 13) = P(X \leq 13) - P(X \leq 7)$

$= 0.8967 - 0.1819$

$= 0.7148$

ii) $E(X) = np = 10$ $\quad Var(X) = npq$

$= 7.5$

Approx with $Y \sim N(\overset{\mu}{10}, \overset{\sigma^2}{\sqrt{7.5}^2})$

$P(8 \leq X \leq 13) \approx P(7.5 \leq Y \leq 13.5)$

≈ 0.7187

Leave blank

2. In a large college 58% of students are female and 42% are male. A random sample of 100 students is chosen from the college. Using a suitable approximation find the probability that more than half the sample are female.
(7)

$X \sim B(\overset{n}{100}, \overset{p}{0.58})$ $E(X) = np = 58$

$Var(X) = npq = 58 \times 0.42$
$= 24.36$

Approximate with

$Y \sim N(58, \sqrt{24.36}^2)$

$P(X \geq 51) \approx P(Y \geq 50.5)$

By calc $= 0.9357$

H 3 2 5 8 1 A 0 6 2 8

Leave blank

5. Sue throws a fair coin 15 times and records the number of times it shows a head.

(a) State the distribution to model the number of times the coin shows a head.
(2)

Find the probability that Sue records

(b) exactly 8 heads,
(2)

(c) at least 4 heads.
(2)

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

(d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly.
(6)

a) $X \sim B(\overset{n}{15}, \overset{p}{0.5})$

b) $P(X = 8) = 0.1964$

c) $P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - 0.0176$
$= \underline{0.9824}$

d) $X \sim B(15, 0.5)$

$H_0 : p = \frac{1}{2}$ $\qquad$ p prob of obtaining a head
$H_1 : p > \frac{1}{2}$

$P(X \geq 13) = 1 - P(X \leq 12)$
$= 1 - 0.9963$
$= 0.0037 < 1\%$

Reject H_0 and accept H_1

There is sufficient evidence to suggest the coin is biased in favour of heads.

H 3 2 5 8 1 A 0 1 4 2 8

Leave blank

3. A single observation x is to be taken from a Binomial distribution B(20, p).

This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as close as possible to 2.5%.
(3)

(b) State the actual significance level of this test.
(2)

The actual value of x obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.
(2)

a) $X \sim B(20, 0.3)$

$P(X \leq 1) = 0.0076$

$P(X \leq 2) = 0.0354$ closest to $2\frac{1}{2}\%$

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.952 \quad = 0.048$

$P(X \geq 11) = 1 - P(X \leq 10)$

$= 1 - 0.9829 \quad = 0.0171$ closest to $2\frac{1}{2}\%$

Critical Region $\{0, 1, 2, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

b) Actual significance level $= 0.0354 + 0.0171$

$= 0.0525 \quad = 5.25\%$

c) 3 is not in critical region

Insufficient evidence to reject H_0. Accept $p = 0.3$

Leave blank

5. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component. **(2)**

(b) Find the probability that there are at least 2 defective components in the box. **(3)**

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components. **(4)**

a) $X \sim B(10, 0.01)$

$P(X=1) = 0.0914$

b) $P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - 0.9957 \quad = 0.0043$

c) Needs Poisson Distribution so omit

Can do with binomial distribution on calculator

$X \sim B(250, 0.01)$

$P(1 \leq X \leq 4) = P(X \leq 4) - P(X \leq 0)$

$= 0.8922 - 0.0811$

$= 0.8111$

Leave blank

1. A bag contains a large number of counters of which 15% are coloured red. A random sample of 30 counters is selected and the number of red counters is recorded.

(a) Find the probability of no more than 6 red counters in this sample.
(2)

A second random sample of 30 counters is selected and the number of red counters is recorded.

(b) Using a Poisson approximation, estimate the probability that the total number of red counters in the combined sample of size 60 is less than 13.
(3)

a) $X \sim B(30, 0.15)$

$P(X \leq 6) = 0.8474$

b) Omit

Leave blank

4. Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's question. You should state the probability of rejection in each tail which should be less than 0.05.

(5)

(b) Write down the actual significance level of a test based on your critical region from part (a).

(1)

The manager found that 11 customers from the sample of 20 had bought baked beans in single tins.

(c) Comment on this finding in the light of your critical region found in part (a).

(2)

a) $X \sim B(20, 0.3)$

$P(X \leq 2) = 0.0354 \quad < 5\%$

$P(X \leq 3) = 0.107$

$P(X \geq 9) = 1 - P(X \leq 8)$

$= 1 - 0.8866 = 0.1134$

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.952 = 0.048 \quad < 5\%$

Critical Region $(X \leq 2) \cup (X \geq 10)$

$P(X \leq 2) = 0.0354, \quad P(X \geq 10) = 0.048$

b) Actual Significance Level $0.0354 + 0.048 = 0.0834$

c) 11 in critical region. Reject H_0 and accept H_1.
Sufficient evidence to suggest proportion of people buying 1 tin has changed from 30%

Leave blank

1. A manufacturer supplies DVD players to retailers in batches of 20. It has 5% of the players returned because they are faulty.

(a) Write down a suitable model for the distribution of the number of faulty DVD players in a batch.

(2)

Find the probability that a batch contains

(b) no faulty DVD players,

(2)

(c) more than 4 faulty DVD players.

(2)

(d) Find the mean and variance of the number of faulty DVD players in a batch.

(2)

a) $X \sim B(20, 0.05)$

b) $P(X=0) = 0.95^{20} = 0.3585$

c) $P(X>4) = 1 - P(X \leq 4) = 1 - 0.9974 = 0.0026$

d) $E(X) = np = 20 \times 0.05 = 1$

$Var(X) = npq = 20 \times 0.05 \times 0.95 = 0.95$

Leave blank

6. (a) Define the critical region of a test statistic. **(2)**

A discrete random variable X has a Binomial distribution B(30, p). A single observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005 **(5)**

(c) Write down the actual significance level of the test. **(1)**

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region. **(2)**

a) Set of values of a test statistic for which null hypothesis is rejected in favour of alternative hypothesis

b)

$X \sim B(30, 0.3)$

$P(x \leq 2) = 0.0021$ closest to 0.005
$P(x \leq 3) = 0.0093$

$P(x \geqslant 16) = 1 - P(x \leq 15)$
$= 1 - 0.9936 = 0.0064$ closest to 0.005
$P(x \geqslant 17) = 1 - P(x \leq 16)$
$= 1 - 0.9978 = 0.0022$

Critical Region $(x \leq 2) \cup (x \geqslant 16)$

c) Actual Significance Level

$= 0.0021 + 0.0064 = 0.0085$

d) 15 not in critical region. Accept Ho, $p = 0.3$

Leave blank

2. Bhim and Joe play each other at badminton and for each game, independently of all others, the probability that Bhim loses is 0.2

Find the probability that, in 9 games, Bhim loses

(a) exactly 3 of the games, **(3)**

(b) fewer than half of the games. **(2)**

Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05

Bhim and Joe agree to play a further 60 games.

(c) Calculate the mean and variance for the number of these 60 games that Bhim loses. **(2)**

(d) Using a suitable approximation calculate the probability that Bhim loses more than 4 games. **(3)**

a) $X \sim B(9, 0.2)$

$P(X=3) = 0.1762$

b) $P(X \leq 4) = 0.9804$

c) $X \sim B(60, 0.05)$ $E(x) = np = 3$

$Var(x) = npq = 3 \times 0.95$

$= 2.85$

d) Needs Poisson so omit

H 3 5 3 9 6 A 0 4 2 4

Leave blank

6. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.
(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025
(3)

(c) Find the actual significance level of this test.
(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer.
(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.
(6)

a) 2 outcomes/faulty or not faulty/success or fail
A constant probability
Independence
Fixed number of trials (fixed n)

b) $X \sim B(50, 0.25)$

$P(X \leq 6) = 0.0193$ closest to $2\frac{1}{2}\%$

$P(X \leq 7) = 0.0452$

$P(X \geq 19) = 1 - P(X \leq 18)$
$= 1 - 0.9712 = 0.0288$ closest to $2\frac{1}{2}\%$

$P(X \geq 20) = 1 - P(X \leq 19)$
$= 1 - 0.986 = 0.014$

Critical Region $(X \leq 6) \cup (X \geq 19)$

H 3 5 3 9 6 A 0 1 6 2 4

Leave blank

Question 6 continued

c) Actual significance level $= 0.0193 + 0.0288$

$= 0.0481$

$= 4.81\%$

d) 8 not in critical region

Accept H_0. Evidence to suggest 25% of bolts are faulty.

e) $X \sim B(50, 0.25)$

$H_0 : p = 0.25$ $\quad$ p = prob bolt is faulty

$H_1 : p < 0.25$

$P(X \leq 5) = 0.007046 < 1\%$

Reject H_0 and accept H_1

There is evidence to suggest that the probability of a bolt being faulty is now less than 25%.