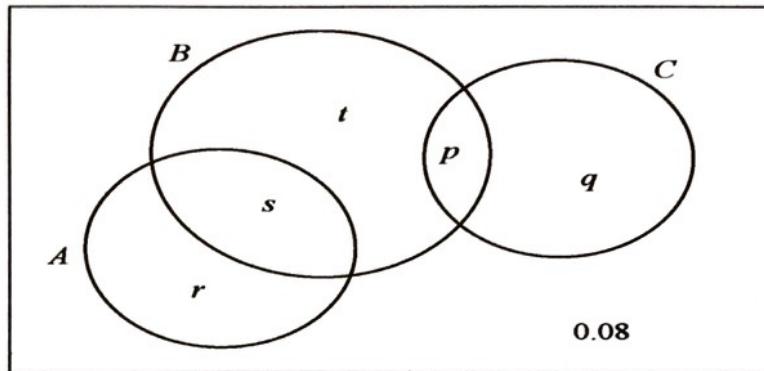


# Venn Diagrams

3. The Venn diagram shows three events  $A$ ,  $B$  and  $C$ , where  $p$ ,  $q$ ,  $r$ ,  $s$  and  $t$  are probabilities.



$P(A) = 0.5$ ,  $P(B) = 0.6$  and  $P(C) = 0.25$  and the events  $B$  and  $C$  are independent.

- (a) Find the value of  $p$  and the value of  $q$ . (2)
- (b) Find the value of  $r$ . (2)
- (c) Hence write down the value of  $s$  and the value of  $t$ . (2)
- (d) State, giving a reason, whether or not the events  $A$  and  $B$  are independent. (2)
- (e) Find  $P(B | A \cup C)$ . (3)

a)  $B, C$  independent  $\Rightarrow P(B \cap C) = P(B) \times P(C)$

$$P = 0.6 \times 0.25$$

$$\underline{P = 0.15}$$

$$q = P(C) - P$$

$$q = 0.25 - 0.15$$

$$\underline{q = 0.1}$$

b)  $r = 1 - 0.08 - q - P(B)$

$$r = 1 - 0.08 - 0.1 - 0.6$$

$$\underline{r = 0.22}$$

c)

$$s = P(A) - r$$

$$s = 0.5 - 0.22$$

$$\underline{s = 0.28}$$

$$t = P(B) - s - r$$

$$t = 0.6 - 0.28 - 0.15$$

$$\underline{t = 0.17}$$

d)

$$P(A) = 0.5 \quad P(B) = 0.6$$

$$P(A \cap B) = s = 0.28$$

$$\text{But } P(A) \times P(B) = 0.5 \times 0.6 = 0.3$$

Since  $0.28 \neq 0.3$

A and B are not independent

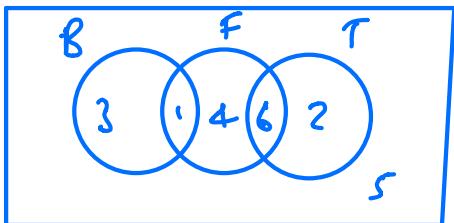
e)

$$P(B | A \cup C) = \frac{P(B \cap (A \cup C))}{P(A \cup C)}$$

$$\begin{aligned} &= \frac{P + s}{P(A) + P(C)} = \frac{0.15 + 0.28}{0.5 + 0.25} \\ &= \frac{0.43}{0.75} = \frac{43}{75} \end{aligned}$$

## Exercise 5C

5)



a) B and T mutually exclusive since  
 $P(B \cap T) = 0$

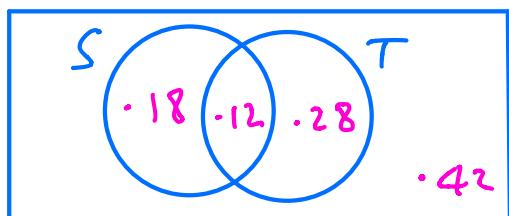
b)  $P(B) = \frac{4}{21}$      $P(F) = \frac{11}{21}$

$$P(B \cap F) = \frac{1}{21} = 0.0476$$

$$P(B) \times P(F) = \frac{4}{21} \times \frac{11}{21} = \frac{44}{441} = 0.09977$$

Not independent since  $0.0476 \neq 0.09977$

7)



a)

$$P(S) = 0.3$$

$$P(T) = 0.4$$

$$P(S \text{ but not } T) = 0.18$$

↓

$$P(S \cap T')$$

$$P(S \cap T) = 0.12$$

$$P(S) \times P(T) = 0.3 \times 0.4 = 0.12$$

$$P(S \cap T) = P(S) \times P(T) = 0.12$$

∴ S, T are independent

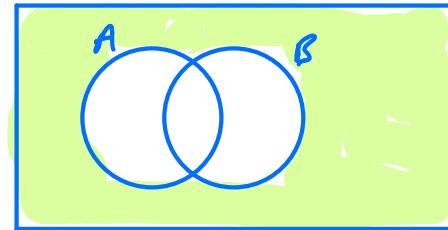
b) i)  $P(S \text{ and } T) = P(S \cap T) = 0.12$

$$\text{ii) } P(S \text{ nor } T) = 0.42$$

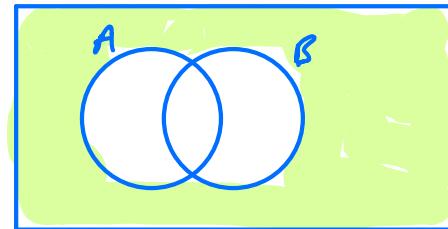
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Point of Interest

Shade  $(A \cup B)'$



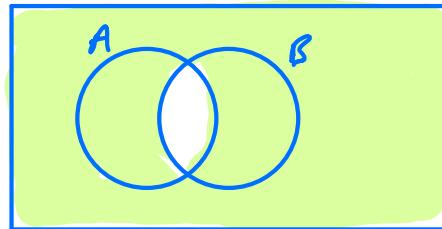
Shade  $A' \cap B'$



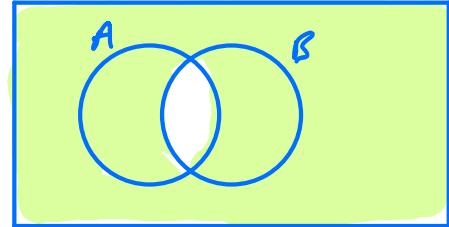
$$\therefore (A \cup B)' = A' \cap B'$$

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Shade  $A' \cup B'$



Shade  $(A \cap B)'$



$$\therefore (A \cap B)' = A' \cup B'$$