Proof Homework Solutions

In all the questions below, n is a positive integer.

17. If $2n$ is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

18. If $(2n + 1)$ is always odd for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive odd numbers cannot be a multiple of 4.

19. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

20. Prove algebraically that the difference between the squares of any two consecutive even numbers is always a multiple of 4.

21. Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

22. Prove algebraically that the difference between the squares of any two consecutive numbers is always an odd number

23. Prove algebraically that the sum of the squares of any three consecutive even numbers always a multiple of 4.

24. Prove algebraically that the sum of the squares of any three consecutive odd numbers always leaves a remainder of 11 when divided by 12.

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18. If $(2n + 1)$ is always odd for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive odd numbers cannot be a multiple of 4.

Let consecutive odd numbers be

$2n-1$ and $2n+3$

$(2n+1)^2 + (2n+3)^2$

$= 4n^2 + 1 + 4n + 4n^2 + 9 + 12n$

$= 8n^2 + 16n + 10$

$= 8n^2 + 16n + 8 + 2$

$= 4(2n^2 + 8n + 2) + 2$

So always 2 more than a multiple of 4

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20. Prove algebraically that the difference between the squares of any two consecutive even numbers is always a multiple of 4.

Let consecutive even numbers be $2n$ and $2n+2$

$(2n+2)^2 - (2n)^2$

$= \cancel{4n^2} + 4 + 8n - \cancel{4n^2}$

$= 8n + 4$

$= 4(2n+1)$ which is a multiple of 4

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22. Prove algebraically that the difference between the squares of any two consecutive numbers is always an odd number

Let consecutive numbers be n and $n+1$

$$(n+1)^2 - n^2$$

$$= \cancel{n^2} + 1 + 2n - \cancel{n^2}$$

$$= 2n+1$$ which is an odd number

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24. Prove algebraically that the sum of the squares of any three consecutive odd numbers always leaves a remainder of 11 when divided by 12.

Let consecutive odd numbers be

$$2n+1, \quad 2n+3, \quad 2n+5$$

$$(2n+1)^2 + (2n+3)^2 + (2n+5)^2$$

$$= 4n^2 + 1 + 4n + 4n^2 + 9 + 12n + 4n^2 + 25 + 20n$$

$$= 12n^2 + 36n + 35$$

$$= 12n^2 + 36n + 24 + 11$$

$$= 12(n^2 + 3n + 2) + 11$$

which is 11 more than a multiple of 12 so leaves a remainder of 11 when divided by 12.
