

Name: \_\_\_\_\_

## Exponential and Log Equations

**Date:**

**Time:**

**Total marks available:**

**Total marks achieved:** \_\_\_\_\_

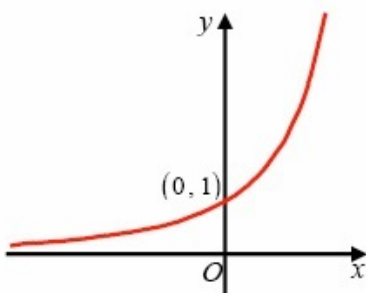
## **Mark Scheme**

Q1.

Question Number	Scheme		Marks
(a)	$2\log(x+15) = \log(x+15)^2$		B1
	$\log(x+15)^2 - \log x = \log \frac{(x+15)^2}{x}$	Correct use of $\log a - \log b = \log \frac{a}{b}$	M1
	$2^6 = 64$ or $\log_2 64 = 6$	64 used in the correct context	B1
	$\log_2 \frac{(x+15)^2}{x} = 6 \Rightarrow \frac{(x+15)^2}{x} = 64$	Removes logs correctly	M1
	$\Rightarrow x^2 + 30x + 225 = 64x$ or $x + 30 + 225x^{-1} = 64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2 - 34x + 225 = 0$ *		A1
			(5)
(b)	$(x-25)(x-9) = 0 \Rightarrow x = 25$ or $x = 9$	M1: Correct attempt to solve the given quadratic as far as $x = \dots$ A1: Both 25 and 9	M1 A1
			(2)
			[7]

Q2.

Question Number	Scheme	Marks
Q (a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$ $y = \frac{1}{8}$ or 0.125	M1 A1 (2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$ [or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$ ] [or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$ ] $\log_2 32 + \log_2 16 = 9$ $(\log x)^2 = \dots$ or $(\log x)(\log x) = \dots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2) $\log_2 x = 3 \Rightarrow x = 2^3 = 8$ $\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	M1  A1 M1 A1 A1ft (5)
(a)	M1 for <u>getting out of logs</u> correctly. If done by change of base, $\log_{10} y = -0.903\dots$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1. A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502\dots$ scores M1 (implied) A0. <u>Correct answer</u> with no working scores both marks. <u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$ .	
(b)	1 <sup>st</sup> M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$ , $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively). 1 <sup>st</sup> A1 for 9 (exact). 2 <sup>nd</sup> M1 for getting $(\log_2 x)^2 = \text{constant}$ . The constant can be a log or a sum of logs. If written as $\log_2 x^2$ instead of $(\log_2 x)^2$ , allow the M mark <u>only</u> if subsequent work implies correct interpretation. 2 <sup>nd</sup> A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0. 3 <sup>rd</sup> A1ft for an answer of $\frac{1}{\text{their } 8}$ . An ft answer may be non-exact. <u>Possible mistakes:</u> $\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = \dots$ scores M1A1(implied by 9)M0A0A0 $\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = \dots$ scores M0A0(9 never seen)M1A0A0 $\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145, x = 0.194$ scores M0A0M1A0A1ft <u>No working</u> (or 'trial and improvement'): $x = 8$ scores M0 A0 M1 A1 A0	

Question Number	Scheme	Marks	
(a)	Graph of $y = 7^x$ , $x \in \mathbb{R}$ and solving $7^{2x} - 4(7^x) + 3 = 0$  At least two of the three criteria correct. (See notes below.) All three criteria correct. (See notes below.)	B1 B1  (2)	
(b)	$y^2 - 4y + 3 = 0$ $\{(y-3)(y-1) = 0 \text{ or } (7^x - 3)(7^x - 1) = 0\}$ $y = 3, y = 1 \text{ or } 7^x = 3, 7^x = 1$ $\{7^x = 3 \Rightarrow\} x \log 7 = \log 3$ $\text{or } x = \frac{\log 3}{\log 7} \text{ or } x = \log_7 3$ $x = 0.5645\dots$ $x = 0$	Forming a quadratic {using "y" = 7^x}. $y^2 - 4y + 3 = 0$  Both $y = 3$ and $y = 1$ . A valid method for solving $7^x = k$ where $k > 0, k \neq 1$  0.565 or awrt 0.56 $x = 0$ stated as a solution.	M1 A1  A1  dM1  A1 B1  (6) [8]
<b>Notes</b>			
(a)	B1B0: Any two of the following three criteria below correct. B1B1: All three criteria correct. Criteria number 1: Correct shape of curve for $x \geq 0$ . Criteria number 2: Correct shape of curve for $x < 0$ . Criteria number 3: (0, 1) stated or 1 marked on the y-axis. Allow (1, 0) rather than (0, 1) if marked in the "correct" place on the y-axis.		

Question Number	Scheme	Marks
(b)	1 <sup>st</sup> M1 is an attempt to form a quadratic equation {using "y" = 7^x}. 1 <sup>st</sup> A1 mark is for the correct quadratic equation of $y^2 - 4y + 3 = 0$ . Can use any variable here, eg: y, x or 7^x. Allow M1A1 for $x^2 - 4x + 3 = 0$ . Writing $(7^x)^2 - 4(7^x) + 3 = 0$ is also sufficient for M1A1. Award M0A0 for seeing $7^{2x} - 4(7^x) + 3 = 0$ by itself without seeing $y^2 - 4y + 3 = 0$ or $(7^x)^2 - 4(7^x) + 3 = 0$ . 1 <sup>st</sup> A1 mark for both $y = 3$ and $y = 1$ or both $7^x = 3$ and $7^x = 1$ . Do not give this accuracy mark for both $x = 3$ and $x = 1$ , unless these are recovered in later working by candidate applying logarithms on these. Award M1A1A1 for $7^x = 3$ and $7^x = 1$ written down with no earlier working. 3 <sup>rd</sup> dM1 for solving $7^x = k, k > 0, k \neq 1$ to give either $x \ln 7 = \ln k$ or $x = \frac{\ln k}{\ln 7}$ or $x = \log_7 k$ . dM1 is dependent upon the award of M1. 2 <sup>nd</sup> A1 for 0.565 or awrt 0.56. B1 is for the solution of $x = 0$ , from <i>any</i> working.	

Q4.

Question number	Scheme	Marks
(a)	$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$ $\log_3 x^2 = 2 \log_3 x$  Using $\log_3 3 = 1$	B1  B1  B1  (3)
(b)	$3x^2 = 28x - 9$  Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$	M1  M1 A1 (3)  <b>6</b>
Notes (a)	<p><b>B1</b> for correct use of addition rule (or correct use of subtraction rule)  <b>B1</b>: replacing <math>\log x^2</math> by <math>2\log x</math> – <b>not</b> <math>\log 3x^2</math> by <math>2\log 3x</math> this is <b>B0</b>  <b>These first two B marks are often earned in the first line of working</b>  <b>B1</b>. for replacing <math>\log 3</math> by 1 (or use of <math>3^1 = 3</math>)            If candidate has been awarded 3 marks and their proof includes an error or omission of reference to <math>\log y</math> withhold the last mark.            So just B1 B1 B0            These marks must be awarded for <b>work in part (a) only</b></p> <p>(b) <b>M1</b> for removing logs to get an equation in <math>x</math>– statement in scheme is sufficient. This needs to be accurate without any errors seen in part (b).  <b>M1</b> for attempting to solve three term quadratic to give <math>x =</math> (see notes on marking quadratics)  <b>A1</b> for the two correct answers – this depends on second M mark only.            Candidates often begin again in part (b) and do not use part (a).            If such candidates make errors in log work in part (b) they score first <b>M0</b>. The second M and the A are earned as before. It is possible to get M0M1A1 or M0M1A0.</p>	
Alternative to (b) using $y$	<p>Eliminates <math>x</math> to give <math>3y^2 - 730y + 243 = 0</math> with no errors is M1            Solves quadratic to find <math>y</math>, then uses values to find <math>x</math> M1            A1 as before</p> <p><b>See extra sheet with examples illustrating the scheme.</b></p>	

Q5.

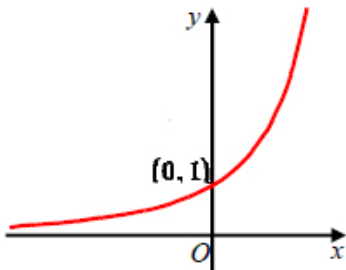
Question Number	Scheme	Marks	
(a)	Either (Way 1) : Attempt $f(3)$ or $f(-3)$ $f(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$ *	Or (Way 2): Assume $a = -9$ and attempt $f(3)$ or $f(-3)$ $f(3) = 0$ so $(x - 3)$ is factor	M1 A1 * cso <b>(2)</b>
	Or (Way 3): $(2x^3 - 5x^2 + ax + 18) \div (x - 3) = 2x^2 + px + q$ where $p$ is a number and $q$ is an expression in terms of $a$ Sets the remainder $18 + 3a + 9 = 0$ and solves to give $a = -9$		M1 A1* cso <b>(2)</b>
(b)	Either (Way 1): $f(x) = (x - 3)(2x^2 + x - 6)$ $= (x - 3)(2x - 3)(x + 2)$		M1A1 M1A1 <b>(4)</b>
	Or (Way 2) Uses trial or factor theorem to obtain $x = -2$ or $x = 3/2$ Uses trial or factor theorem to obtain both $x = -2$ and $x = 3/2$ Puts three factors together (see notes below) Correct factorisation : $(x - 3)(2x - 3)(x + 2)$ or $(3 - x)(3 - 2x)(x + 2)$ or $2(x - 3)(x - \frac{3}{2})(x + 2)$ oe		M1 A1 M1 A1 <b>(4)</b>
	Or (Way 3) No working three factors $(x - 3)(2x - 3)(x + 2)$ otherwise need working		M1A1M1A1
(c)	$\{3^y = 3 \Rightarrow\} \underline{y = 1}$ or $g(1) = 0$		B1
	$\{3^y = 1.5 \Rightarrow\} \log(3^y) = \log 1.5$ or $y = \log_3 1.5$		M1
	$\{y = 0.3690702\dots\} \Rightarrow y = \text{awrt } 0.37$		A1 (3)
		<b>[9]</b>	
<b>Notes for Question</b>			
(a)	M1 for attempting either $f(3)$ or $f(-3)$ – with numbers substituted into expression A1 for applying $f(3)$ correctly, setting the result equal to 0, and manipulating this correctly to give the result given on the paper i.e. $a = -9$ . (Do not accept $x = -9$ ) Note that the answer is given in part (a). If they assume $a = -9$ and verify by factor theorem or division they must state $(x - 3)$ is a factor for A1 (or equivalent such as QED or a tick).		
(b)	1 <sup>st</sup> M1: attempting to divide by $(x - 3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$ . (Could divide by $(3 - x)$ , in which case the quadratic would begin $-2x^2$ .) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. 1 <sup>st</sup> A1: usually for $2x^2 + x - 6$ ... Credit when seen and use isw if miscopied 2 <sup>nd</sup> M1: for a valid* attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2 <sup>nd</sup> A1 is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) NB: $(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M0A0, $(x - 3)(x - \frac{3}{2})(2x + 4)$ is M1A1M1A0, but $2(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M1A1.		
(c)	B1: $\underline{y = 1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for $g(1) = 0$ . M1: Attempt to take logs to solve $3^y = \alpha$ or even $3^{ky} = \alpha$ , but not $6^y = \alpha$ where $\alpha > 0$ and $\alpha \neq 3$ & was a root of $f(x) = 0$ (ft their factorization) A1: for an answer that rounds to 0.37. If a third answer is included (and not “rejected”) such as $\ln(-2)$ lose final A mark		

Question Number	Scheme	Marks
<b>(i)</b> <b>Method 1</b>	$\log_2\left(\frac{2x}{5x+4}\right) = -3$ or $\log_2\left(\frac{5x+4}{2x}\right) = 3$ , or $\log_2\left(\frac{5x+4}{x}\right) = 4$ (see special case 2)	M1
	$\left(\frac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\frac{5x+4}{2x}\right) = 2^3$ or $\left(\frac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\frac{2x}{5x+4}\right)\right) = \log_2\left(\frac{1}{8}\right)$	M1
	$16x = 5x + 4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent)	dM1
	$x = \frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	A1 cso <b>(4)</b>
<b>(i)</b> <b>Method 2</b>	$\log_2(2x) + 3 = \log_2(5x + 4)$	
	So $\log_2(2x) + \log_2(8) = \log_2(5x + 4)$ (3 replaced by $\log_2 8$ )	2 <sup>nd</sup> M1
	Then $\log_2(16x) = \log_2(5x + 4)$ (addition law of logs)	1 <sup>st</sup> M1
	Then final M1 A1 as before	dM1A1
<b>(ii)</b>	$\log_a y + \log_a 2^3 = 5$	M1
	$\log_a 8y = 5$	Applies product law of logarithms. dM1
	$y = \frac{1}{8}a^5$	$y = \frac{1}{8}a^5$ A1cao <b>(3)</b> [7]
<b>Notes for Question</b>		
<b>(i)</b>	1 <sup>st</sup> M1: Applying the subtraction or addition law of logarithms correctly to make two log terms in $x$ into one log term in $x$	
	2 <sup>nd</sup> M1: For RHS of either $2^{-3}$ , $2^3$ , $2^4$ or $\log_2\left(\frac{1}{8}\right)$ , $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an earlier error. Use of $3^2$ is M0	
	3 <sup>rd</sup> dM1: Obtains correct linear equation in $x$ . usually the one in the scheme and attempts $x =$	
	A1: cso Answer of $4/11$ with no suspect log work preceding this.	
<b>(ii)</b>	M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ or $\log_a 8$	
	dM1: (should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$	
<b>(i)</b>	<b>Special case 1:</b> $\log_2(2x) = \log_2(5x + 4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x + 4)} = -3 \Rightarrow \frac{2x}{5x + 4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ or	
	$\log_2(2x) = \log_2(5x + 4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x + 4)} = -3 \Rightarrow \log_2 \frac{2x}{5x + 4} = -3 \Rightarrow \frac{2x}{5x + 4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ each	
	attempt scores M0M1M1A0 – special case	
	<b>Special case 2:</b>	
	$\log_2(2x) = \log_2(5x + 4) - 3 \Rightarrow \log_2 2 + \log_2 x = \log_2(5x + 4) - 3$ , is M0 until the two log terms are combined to give $\log_2\left(\frac{5x+4}{x}\right) = 3 + \log_2 2$ . This earns M1	
	Then $\left(\frac{5x+4}{x}\right) = 2^4$ or $\log_2\left(\frac{5x+4}{x}\right) = \log_2 2^4$ gets second M1. Then scheme as before.	

Question Number	Scheme		Marks
(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ $= 2 + a$	or Way 2: $\log_3(9x) = \log_3 3^{a+2}$ $= 2 + a$	M1 A1 (2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ $\log x^5 = 5 \log x$ or $\log 81 = 4 \log 3$ or $\log 81 = 4$ $= 5a - 4$	or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$ $= \log_3 3^{5a-4}$	M1 M1 A1 cso (3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$ Method 1 $\Rightarrow 2 + a + 5a - 4 = 3$ $\Rightarrow a = \frac{5}{6}$ $\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a \log_{10} 3$ so $x =$ $x = 2.498$ or awrt If $x = -2.498$ appears as well or instead this is A0	Method 2 $\log_3\left(9x \cdot \frac{x^5}{81}\right) = (3 \text{ or } \log 27)$ $\log_3\left(\frac{x^6}{9}\right) = 3 \text{ or } \log 27$ $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$ $x = 2.498$ or awrt	M1 A1 M1 A1 (4) <b>Total 9</b>
<b>Notes for Question</b>			
(a)	Way 1: M1: Use of $\log(ab) = \log(a) + \log(b)$ A1: must be $a + 2$ or $2 + a$ Way 2: Uses $x = 3^a$ to give $\log_3(9x) = \log_3 3^{a+2}$ , A1 for $a + 2$ or $2 + a$		
(b)	Way 1: M1: Use of $\log(a/b) = \log(a) - \log(b)$ M1: Use of $n \log(a) = \log(a)^n$ Way 2: M1 Use of correct powers of 3 in numerator and denominator M1: Subtracts powers A1: No errors seen		
(c)	<b>Method 1:</b> M1: Uses (a) and (b) results to form an equation in $a$ (may not be linear) A1: $a =$ awrt 0.833 M1: Finds $x$ by use of 3 to a power, or change of base performed correctly A1: $x = 2.498$ (accept answer which round to this value from 2.498049533...) <b>Method 2:</b> M1: Use of $\log(ab) = \log(a) + \log(b)$ in an equation (RHS may be wrong) A1: Equation correct and simplified M1: Tries to undo log by 3 to power correctly, and uses root to obtain $x$ A1: $x = 2.498$ (accept answer which round to this value from 2.498049533...) Lose this mark if negative answer is given as well as or instead of positive answer.		



Q8.

Question Number	Scheme		Marks	
	Graph of $y = 3^x$ and solving $3^{2x} - 9(3^x) + 18 = 0$			
(a)		At least two of the three criteria correct. (See notes below.)	B1	
		All three criteria correct. (See notes below.)	B1	
		<b>Criteria number 1:</b> Correct shape of curve for $x \geq 0$ and at least touches the positive $y$ -axis. <b>Criteria number 2:</b> Correct shape of curve for $x < 0$ . Must not touch the $x$ -axis or have any turning points. <b>Criteria number 3:</b> $(0, 1)$ stated or in a table or 1 marked on the $y$ -axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the "correct" place on the $y$ -axis.		
			[2]	
(b)	$(3^x)^2 - 9(3^x) + 18 = 0$ or $y = 3^x \Rightarrow y^2 - 9y + 18 = 0$	Forms a quadratic of the correct form in $3^x$ or in " $y$ " where " $y$ " = $3^x$ or even in $x$ where " $x$ " = $3^x$	M1	
	$\{(y-6)(y-3) = 0 \text{ or } (3^x-6)(3^x-3) = 0\}$			
	$y = 6, y = 3 \text{ or } 3^x = 6, 3^x = 3$	Both $y = 6$ and $y = 3$ .	A1	
	$\{3^x = 6 \Rightarrow\} x \log 3 = \log 6$ or $x = \frac{\log 6}{\log 3}$ or $x = \log_3 6$	A valid method for solving $3^x = k$ where $k > 0, k \neq 1, k \neq 3$ to give either $x \log 3 = \log k$ or $x = \frac{\log k}{\log 3}$ or $x = \log_3 k$	dM1	
	$x = 1.63092\dots$	awrt 1.63	A1cso	
	Provided the first M1A1 is scored, the second M1A1 can be implied by awrt 1.63			
	$x = 1$	$x = 1$ stated as a solution from <i>any</i> working.	B1	
			[5]	
			<b>Total 7</b>	

Q9.

Question Number	Scheme		Marks
(i)	$5^y = 8$		
	$y \log 5 = \log 8$	$y \log 5 = \log 8$ or $y = \log_5 8$	M1
	$\left\{ y = \frac{\log 8}{\log 5} \right\} = 1.2920\dots$	awrt 1.29	A1
	<b>Allow correct answer only</b>		
			<b>[2]</b>
(ii)	$\log_2(x + 15) - 4 = \frac{1}{2} \log_2 x$		
	$\log_2(x + 15) - 4 = \log_2 x^{\frac{1}{2}}$	Applies the power law of logarithms seen <b>at any point in their working</b>	M1
	$\log_2\left(\frac{x + 15}{x^{\frac{1}{2}}}\right) = 4$	Applies the subtraction or addition law of logarithms <b>at any point in their working</b>	M1
	$\left(\frac{x + 15}{x^{\frac{1}{2}}}\right) = 2^4$	Obtains a correct expression with logs removed and no errors	M1
	$x - 16x^{\frac{1}{2}} + 15 = 0$ or e.g. $x^2 + 225 = 226x$	Correct <b>three term quadratic</b> in any form	A1
	$(\sqrt{x} - 1)(\sqrt{x} - 15) = 0 \Rightarrow \sqrt{x} = \dots$	A <b>valid</b> attempt to factorise or solve their <b>three term quadratic</b> to obtain $\sqrt{x} = \dots$ or $x = \dots$ Dependent on all previous method marks.	dddM1
	$\{\sqrt{x} = 1, 15\}$		
	$x = 1, 225$	<b>Both, ignore any other values of <math>x \leq 0</math> from an otherwise correct solution)</b>	A1
			<b>[6]</b>
		<b>Total 8</b>	
<b>Alternative:</b>			
	$2 \log_2(x + 15) - 8 = \log_2 x$		
	$\log_2(x + 15)^2 - 8 = \log_2 x$	Applies the power law of logarithms	M1
	$\log_2\left(\frac{(x + 15)^2}{x}\right) = 8$	Applies the subtraction law of logarithms	M1
	$\frac{(x + 15)^2}{x} = 2^8$	Obtains a correct expression with logs removed	M1
	$x^2 + 30x + 225 = 256x$		
	$x^2 - 226x + 225 = 0$	Correct <b>three term quadratic</b> in any form	A1
	$(x - 1)(x - 225) = 0 \Rightarrow x = \dots$	A <b>valid</b> attempt to factorise or solve their <b>3TQ</b> to obtain $x = \dots$ Dependent on all previous method marks.	dddM1
	$x = 1, 225$	<b>Both <math>x = 1</math> and <math>x = 225</math> (If both are seen, ignore any other values of <math>x \leq 0</math> from an otherwise correct solution)</b>	A1

Q10.

Question Number	Scheme	Marks
(i)	$8^{2x+1} = 24$ $(2x+1)\log 8 = \log 24 \text{ or } (2x+1) = \log_8 24$ $x = \frac{1}{2} \left( \frac{\log 24}{\log 8} - 1 \right) \text{ or } x = \frac{1}{2} (\log_8 24 - 1)$ $= 0.264$ <div style="display: inline-block; vertical-align: middle; border-left: 1px solid black; padding-left: 10px; margin-left: 20px;"> <math display="block">\text{or } 8^{2x} = 3 \text{ and so } (2x)\log 8 = \log 3 \text{ or } (2x) = \log_8 3</math> <math display="block">x = \frac{1}{2} \left( \frac{\log 3}{\log 8} \right) \text{ or } x = \frac{1}{2} (\log_8 3) \text{ o.e.}</math> </div>	<p>M1</p> <p>dM1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
(ii)	$\log_2 (11y - 3) - \log_2 3 - 2\log_2 y = 1$ $\log_2 (11y - 3) - \log_2 3 - \log_2 y^2 = 1$ $\log_2 \frac{(11y - 3)}{3y^2} = 1 \quad \text{or} \quad \log_2 \frac{(11y - 3)}{y^2} = 1 + \log_2 3 = 2.58496501$ $\log_2 \frac{(11y - 3)}{3y^2} = \log_2 2 \quad \text{or} \quad \log_2 \frac{(11y - 3)}{y^2} = \log_2 6 \text{ (allow awrt 6 if replaced by 6 later)}$ <p>Obtains <math>6y^2 - 11y + 3 = 0</math> o.e. i.e. <math>6y^2 = 11y - 3</math> for example</p> <p>Solves quadratic to give <math>y =</math></p> <p><math>y = \frac{1}{3}</math> and <math>\frac{2}{3}</math> (need both- one should not be rejected)</p>	<p>M1</p> <p>dM1</p> <p>B1</p> <p>A1</p> <p>ddM1</p> <p>A1</p> <p style="text-align: right;">(6)</p>
Notes (i)	<p><b>M1:</b> Takes logs and uses law of powers correctly. (Any log base may be used) Allow lack of brackets.</p> <p><b>dM1:</b> Make <math>x</math> subject of their formula correctly (may evaluate the log before subtracting 1 and calculate e.g. <math>(1.528 - 1)/2</math>)</p> <p><b>A1:</b> Allow answers which round to 0.264</p> <p>(ii) <b>M1:</b> Applies power law of logarithms replacing <math>2\log_2 y</math> by <math>\log_2 y^2</math></p> <p><b>dM1:</b> Applies quotient or product law of logarithms correctly to the three log terms including term in <math>y^2</math>. (dependent on first M mark) or applies quotient rule to two terms and collects constants (allow "triple" fractions) <math>1 + \log_2 3</math> on RHS is not sufficient – need <math>\log_2 6</math> or 2.58...</p> <p>e.g. <math>\log_2 (11y - 3) = \log_2 3 + \log_2 y^2 + \log_2 2</math> becoming <math>\log_2 (11y - 3) = \log_2 6y^2</math></p> <p><b>B1:</b> States or uses <math>\log_2 2 = 1</math> or <math>2^1 = 2</math> at any point in the answer so may be given for</p> <p><math>\log_2 (11y - 3) - \log_2 3 - 2\log_2 y = \log_2 2</math> or for <math>\frac{(11y - 3)}{3y^2} = 2</math>, for example (Sometimes this mark will be awarded before the second M mark, and it is possible to score M1M0B1 in some cases)</p> <p>Or may be given for <math>\log_2 6 = 2.584962501..</math> or <math>2^{2.584962501..} = 6</math></p> <p><b>A1:</b> This or equivalent quadratic equation (does not need to be in this form but should be equation)</p> <p><b>ddM1:</b> (dependent on the two previous M marks) Solves their quadratic equation following reasonable log work using factorising, completion of square, formula or implied by both answers correct.</p> <p><b>A1:</b> Any equivalent correct form – need both answers- allow awrt 0.333 for the answer <math>1/3</math></p> <p>*NB: If "=0" is missing from the equation but candidate continues correctly and obtains correct answers then allow the penultimate A1 to be implied (Allow use of <math>x</math> or other variable instead of <math>y</math> throughout)</p>	