Name: _____

Elastic Collisions

Date:

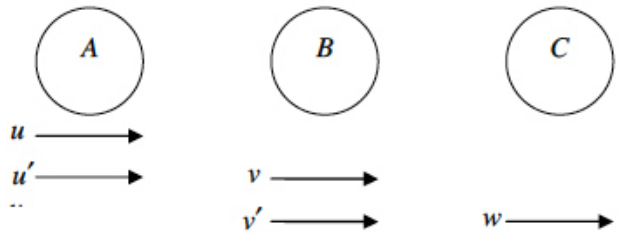
Time:

Total marks available:

Total marks achieved: _____

Mark Scheme

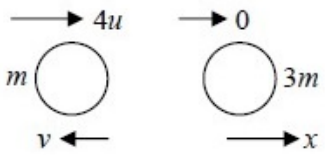
Q1.

Question Number	Scheme	Marks
(a)		
	Momentum: $u = u' + v$	M1 A1
	NEL: $v - u' = eu$	M1 A1
	$2v = u(1 + \frac{2}{3}),\ v = \frac{1}{2}u \times \frac{5}{3} = \frac{5u}{6}$	DM1 A1
	$u' = u - v = \frac{u}{6}$	A1
		(7)
(b)	KE lost $= \frac{1}{2}mu^2 - \left(\frac{1}{2}m \times \frac{25}{36}u^2 + \frac{1}{2}m \times \frac{1}{36}u^2\right)$ their speeds	M1
	$= \frac{1}{2}mu^2 - \left(\frac{1}{2}m \times \frac{26}{36}u^2\right)$	A2 – 1ee
	$= \frac{1}{2}mu^2 \times \frac{10}{35} = \frac{5}{36}mu^2$ **AG**	A1
		(4)
(c)	Speed of C $= \frac{1}{2}\left(\frac{1}{2}u\left(\frac{5}{3}\right)\right)\left(\frac{5}{3}\right) = \frac{1}{2} \cdot \frac{5u}{6} \cdot \frac{5}{3}, = \frac{25}{36}u$	M1 A1 DM1 A1
		(4) **15**

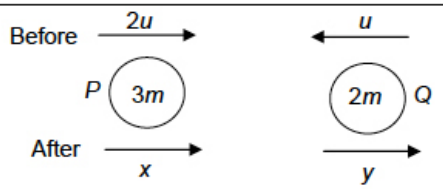
Q2.

Question Number	Scheme	Marks	Notes
(a)	$3m.2u - 4mu = 3mv_1 + 4mv_2$	M1 A1	CLM. Need all terms. Condone sign slips. Correct but check their directions for v_1 & v_2.
	$e(2u+u) = -v_1 + v_2$	M1 A1	Impact law. Must be used the right way round, but condone sign slips. Directions of v_1 & v_2 must be consistent between the two equations. (Ignore the diagram if necessary)
	$\frac{u(2+9e)}{7} = v_2$	DM1 A1 (6)	Eliminate v_1 to produce an equation in v_2 only. Dependent on both previous M marks – must be using both equations. DO NOT accept the negative. The question asks for speed.
(b)	$v_1 = \frac{2u(1-6e)}{7}$	M1 A1	Use the work from (a) or restart to find v_1 or λv_1 for a constant λ. If using work from (a) this mark is dependent on the first 2 M marks. a.e.f. Correct for their direction. Allow for λv_1
	$v_1 < 0 \Rightarrow e > \frac{1}{6}$	DM1 A1	An appropriate inequality for their v_1 (seen or implied) – requires previous M1 scored. Work on v_1=0 scores M0 until the inequality is formed. Accept $\frac{2}{12}$. Answer must follow from correct work for v_1
	$1 \geq e > \frac{1}{6}$	B1 (5)**11**	For (their value) $< e \leq 1$ SR: from $v_1 \leq 0$ could score M1A0B1

Q3.

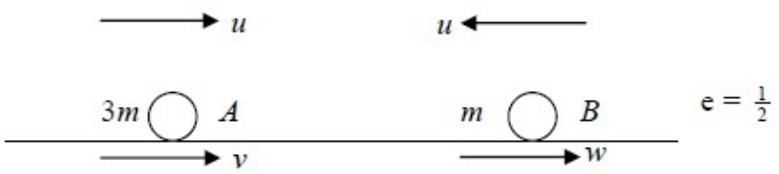
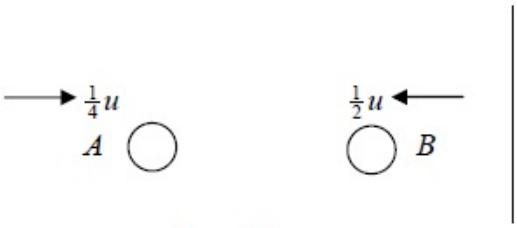
Question Number	Scheme	Marks
		
	$4mu = 3mx - mv$	M1 A1
	$4ue = x + v$	M1 A1
	$4u = 3(4ue - v) - v$ $4u = 12ue - 4v$	
	$v = (3e-1)u$	DM1 A1
	$v > 0 \Rightarrow 3e > 1$	DM1
	$\therefore e > \frac{1}{3}$ **	A1
		(8) **8**

Q4.

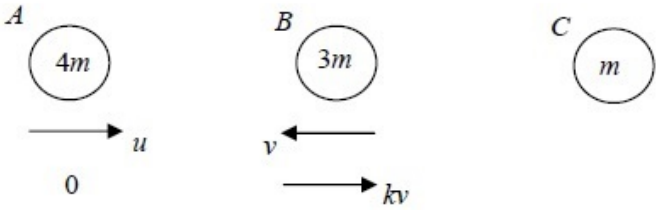
Question Number	Scheme	Marks
(a)	 Correct use of NEL	M1*
	$y - x = e(2u + u)$ o.e.	A1
	CLM $(\rightarrow)$: $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ $(\Rightarrow 4u = 3x + 2y)$	B1
	Hence $x = y - 3eu$, $4u = 3(y-3eu) + 2y$, $(u(9e+4) = 5y)$	d*M1
	Hence, speed of $Q = \underline{\frac{1}{5}(9e+4)u}$ **AG**	A1 cso
		(5)
(b)	$x = y - 3eu = \frac{1}{5}(9e+4)u - 3eu$	M1#
	Hence, speed P $= \frac{1}{5}(4-6e)u = \frac{2u}{5}(2-3e)$ o.e.	A1
	$x = \frac{1}{2}u = \frac{2u}{5}(2-3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e	d#M1
	gives, $e = \frac{3}{12} \Rightarrow \underline{e = \frac{1}{4}}$ **AG**	A1
		(4)
Or (b)	Using NEL correctly with given speeds of P and Q	M1#
	$3eu = \frac{1}{5}(9e+4)u - \frac{1}{2}u$	A1
	$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e	d#M1
	$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.	A1
		(4)
(c)	Time taken by Q from A to the wall $= \frac{d}{\underline{y}} = \left\{\frac{4d}{5u}\right\}$	M1†
	Distance moved by P in this time $= \underline{\frac{u}{2} \times \frac{d}{y}}$ $\left(= \frac{u}{2}\left(\frac{4d}{5u}\right) = \frac{2}{5}d\right)$	A1
	Distance of P from wall $= d - x\left(\frac{d}{y}\right); = d - \frac{2}{5}d = \underline{\frac{3}{5}d}$ **AG**	d†M1; A1 cso
		(4)
or (c)	Ratio speed P:speed Q = x:y $= \frac{1}{2}u : \frac{1}{5}(\frac{9}{4}+4)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$	M1†
	So if Q moves a distance d, P will move a distance $\frac{2}{5}d$	A1
	Distance of P from wall $= d - \frac{2}{5}d; = \underline{\frac{3}{5}d}$ **AG** cso	d†M1; A1
		(4)

Question Number	Scheme	Marks
(d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft
	Time for P, $T_{AB} = \frac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \frac{x}{\frac{1}{4}u}$ from their y	B1ft
	Hence $T_{AB} = T_{WB} \Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u}$	M1
	gives, $2\left(\frac{3d}{5} - x\right) = 4x \Rightarrow \frac{3d}{5} - x = 2x$, $3x = \frac{3d}{5} \Rightarrow x = \underline{\frac{1}{5}d}$	A1 cao
		(4)
or (d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft
	speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their y	B1ft
	Distance of B from wall $= \frac{1}{3} \times \frac{3d}{5}; = \frac{d}{5}$ their $\frac{1}{2+1}$	M1; A1
		(4)
2nd or (d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft
	Combined speed of P and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$	
	Time from wall to 2nd collision $= \frac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y	B1ft
	Distance of B from wall = (their speed)x(their time) $= \frac{u}{4} \times \frac{4d}{5u}; = \underline{\frac{1}{5}d}$	M1; A1
		(4)
		[17]

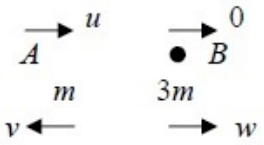
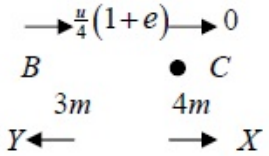
Q5.

Question Number	Scheme	Marks
(a) (i)		
	Con. of Mom: $3mu - mu = 3mv + mw$	
	$2u = 3v + w$ (1)	M1# A1
	N.L.R: $\frac{1}{2}(u+u) = w - v$	M1# A1
	$u = w - v$ (2)	
	(1) – (2) $u = 4v$	DM1#
	$v = \frac{1}{4}u$	A1
(ii)	In (2) $u = w - \frac{1}{4}u$	
	$w = \frac{5}{4}u$	A1 (7)
(b)	B to wall: N.L.R: $\frac{5}{4}u \times \frac{2}{5} = V$	M1
	$V = \frac{1}{2}u$	A1ft (2)
(c)		
	B to wall: time $= 4a \div \frac{5}{4}u = \frac{16a}{5u}$	B1ft
	Dist. Travelled by $A = \frac{1}{4}u \times \frac{16a}{5u} = \frac{4}{5}a$	B1ft
	In t secs, A travels $\frac{1}{4}ut$, B travels $\frac{1}{2}ut$	
	Collide when speed of approach $= \frac{1}{2}ut + \frac{1}{4}ut$, distance to cover = $4a - \frac{4}{5}a$	M1$
	$\therefore\ t = \frac{4a - \frac{4}{5}a}{\frac{3}{4}u} = \frac{16a}{5} \times \frac{4}{3u} = \frac{64a}{15u}$	DM1$ A1
	Total time $= \frac{16a}{5u} + \frac{64a}{15u} = \frac{112a}{15u}$ *	A1 (6)
		15

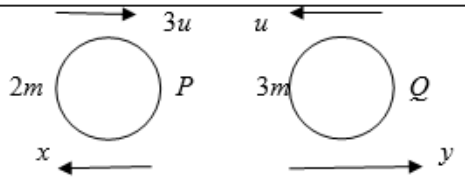
Q6.

Question Number	Scheme	Marks
(a)		
	Conservation of momentum: $4mu - 3mv = 3mkv$	M1A1
	Impact law: $kv = \frac{3}{4}(u+v)$	M1A1
	Eliminate k: $4mu - 3mv = 3m \times \frac{3}{4}(u+v)$	DM1
	$u = 3v$ (Answer given)	A1
		(6)
(b)	$kv = \frac{3}{4}(3v+v)$, $k = 3$	M1,A1
		(2)
(c)	Impact law: $(kv+2v)e = v_C - v_B$ $(5ve = v_C - v_B)$	B1
	Conservation of momentum : $3 \times kv - 1 \times 2v = 3v_B + v_c$ $(7v = 3v_B + v_c)$	B1
	Eliminate v_C : $v_B = \frac{v}{4}(7-5e) > 0$ hence no further collision with A.	M1 A1
		(4)
		[12]

Q7.

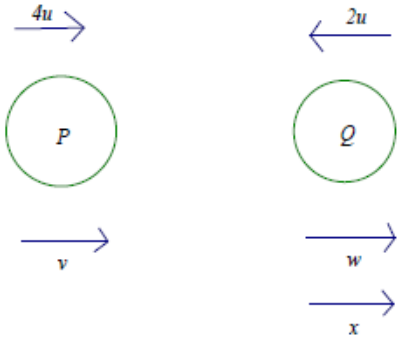
Q	Scheme	Marks	
(a)			If the signs on their diagram and in their working are inconsistent, ignore the diagram. Penalise inconsistency between the two equations in the second accuracy mark.
	$mu = -mv + 3mw$	M1	CLM. Allow for v in either direction. Needs all 3 terms. Condone sign errors.
	$u = -v + 3w$	A1	v in either direction. Ignore diagram if equations "correct" but inconsistent with diagram.
	$eu = w + v$	M1	Impact law. Must be the right way round, but condone sign errors
		A1	Correct equation. Signs consistent with CLM equn.
	$w = \frac{u}{4}(1+e)$	DM1	Solve for v or w.
		A1	One correct
	$v = -w + eu = \frac{u}{4}(3e-1)$	A1	Both correct. $1-3e \to$ A0 for v
(b)			If the signs on their diagram and in their working are inconsistent, ignore the diagram. Penalise inconsistency between the two equations in the B mark.
	$3mw = 4mX - 3mY$	M1	CLM for their w.
		A1ft	Correct unsimplified (their w)
	$2ew = X + Y$	B1ft	Impact law. Must be the right way up. Their w
	$7Y = W(8e-3)$	DM1	Solve for (7)Y
	Or $2ue(1+e) - \frac{3u}{4}(1+e) = 7Y$		
	$\to e > \frac{3}{8}$	A1	
	$Y > 0 \to \frac{3}{8} < e \le \frac{1}{2}$	A1	NB No longer ft. Condone <.
(c)	$\frac{u}{28}(1+e)(8e-3) > \frac{u}{4}(3e-1)$	M1	For a second collision their $Y >$ their v
	$2e^2 - 4e + 1 > 0$		
	$e = \frac{4 \pm \sqrt{16-8}}{4} = 1.707,\ 0.293$	DM1	Obtain the critical values
	$2e^2 - 4e + 1 < 0$ for $\frac{3}{8} < e \le \frac{1}{2}$ so no second collision.	A1	Compare 0.293 (o.e.) with $\frac{3}{8}$ to reach correct conclusion for correct reason.

Q8.

Question Number	Scheme	Marks	Notes
			
(a)	$6mu - 3mu = 3my - 2mx$	M1	CLM Needs all the terms. Condone sign errors
	$3y - 2x = 3u$	A1	Correct equation
	$4ue = x + y$	M1	Impact law. Must be used the right way round.
		A1	Correct equation. Signs with x, y must be consistent with the CLM equation.
	$y = \frac{u}{5}(8e+3)$ **	DM1	Dependent on the two preceding M marks
		A1 (6)	Obtain the **given result** correctly
	This is a given result – the candidate needs to show sufficient working to support the answer.		
(b)	$x = 4ue - y$	M1	Use one of their equations from (a) and $\pm$ the given y to find x.
	$x = \frac{1}{5}u(20e - 8e - 3) = \frac{3}{5}u(4e-1)$	A1	Any equivalent form. Accept $\pm$
	$x > 0 \Rightarrow e > \frac{1}{4}$	M1	Use x > 0 to solve find values of e. The inequality must match their x.
	$\therefore \frac{1}{4} < e \leqslant 1$	A1 (4)	Need both limits.

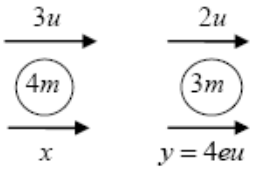
(c)	$e = \frac{1}{2} \quad x = \frac{3u}{5} \quad y = \frac{u}{5}(4+3) = \frac{7u}{5}$	B1	Allow $x = -\frac{3u}{5}$
	$T = \frac{1}{2} \times 2m \times 9u^2 + \frac{1}{2} \times 3mu^2 \quad \left(= \frac{21mu^2}{2}\right)$	M1	KE before or KE after, in terms of u
	$kT = \frac{1}{2} \times 2m \times \frac{9u^2}{25} + \frac{1}{2} \times 3m \times \frac{49u^2}{25} \quad \left(= \frac{165mu^2}{50}\right)$	M1	Second KE in terms of u and use to find k.
	$k = \dfrac{\frac{1}{2} \times 2 \times \frac{9}{25} + \frac{1}{2} \times 3 \times \frac{49}{25}}{\frac{1}{2} \times 2 \times 9 + \frac{1}{2} \times 3} \quad \left(\text{or } \frac{165}{50} \div \frac{21}{2}\right) = \frac{11}{35}$	A1 (4)	Or equivalent. 0.314 or better
		[14]	

Q9.

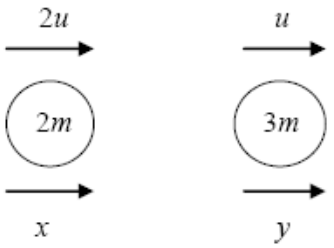
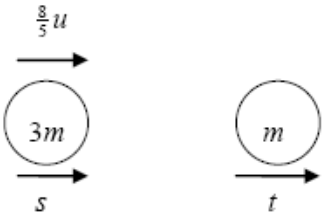
Question Number	Scheme	Marks	
a			
	$4mu - 2mu = mv + mw$	M1	Equation for CLM. Requires all 4 terms. Condone sign errors. Condone m missing throughout.
		A1	
	$w - v = 6eu$	M1	Impact law. e must be used correctly. Condone sign errors
		A1	Signs should be consistent with equation for CLM.
	$v + w = 2u$ $w - v = 6eu$ $\quad 2w = 2u + 6eu$, $(w = u + 3eu)$	DM1	Solve for w. Dependent on the two previous M marks.
	For Q and R to collide require $w > 3u$,	M1	Use **inequality** to compare their w with $3u$.
	$u + 3eu > 3u$, $e > \frac{2}{3}$	A1	Reach ***Given answer*** with no errors seen.
		[7]	

a alt	Collision between Q and $R \Rightarrow w > 3u$	M1	
	Magnitude of impulse on $Q > 5mu$		
	Magnitude of impulse on $P > 5mu$	A1	
	$\Rightarrow v < -u$	M1	
	$e = \dfrac{w-v}{4u+2u}$	M1	Impact law
	Speed of separation after collision $> u + 3u$	M1	
	$e > \dfrac{4u}{4u+2u}$	A1	
	$e > \dfrac{2}{3}$	A1	Reach given inequality with no errors seen.
b	$w = u + 3eu = \dfrac{13}{4}u$, $v = -\dfrac{5}{4}u$	B1	Correct values seen or implied.
	$mw + 3mu = mx + my$ $\dfrac{3}{4}(w - 3u) = y - x$	M1	Equations for CLM and impact. Allow with w or their w. All terms required. Condone sign errors. e must be used correctly.
		A1	Correct equations in w or their w.
	$\dfrac{25}{4}u = x + y$, $\dfrac{3u}{16} = y - x$		
	Solve for x (or kx)	DM1	Dependent on the previous M1 Need to get far enough to give a convincing concluding argument.
	$\dfrac{97}{16}u = 2x$, $x = \dfrac{97u}{32}(= 3.03125u)$	A1	Correct expression for kx
	P and Q moving away from each other, so no collision.	A1	Reach **given conclusion** with no errors seen.
		[6]	
		(13)	

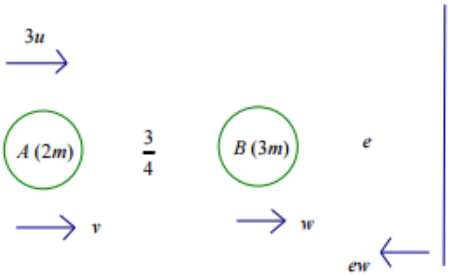
Q10.

Question Number	Scheme	Marks
(*a*)		
	LM $12mu + 6mu = 4mx + 12meu$	B1
	NEL $4eu - x = eu$	M1 A1
	Eliminating x to obtain equation in e	DM1
	Leading to $e = \frac{3}{4}$ ✱ cso	A1 cso (5)
(*b*)	$x = 3eu$ or $\frac{9}{4}u$ or $4.5u - 3eu$ seen or implied	B1
	Loss in KE $= \frac{1}{2}4m(3u)^2 + \frac{1}{2}3m(2u)^2 - \frac{1}{2}4m\left(\frac{9}{4}u\right)^2 - \frac{1}{2}3m(3u)^2$	M1 A1ft
	$= 24mu^2 - 23\frac{5}{8}mu^2 = \frac{3}{8}mu^2 = 0.375mu^2$	A1 (4)
		(9 marks)

Q11.

Question Number	Scheme	Marks
	(a) 	
	LM $4mu + 3mu = 2mx + 3my$	M1 A1
	NEL $y - x = \frac{1}{2}u$	B1
	Solving to $y = \frac{8}{5}u$ ✱ cso	M1 A1 **(5)**
	(b) $x = \frac{11}{10}u$ or equivalent	B1
	Energy loss $\frac{1}{2}\times 2m\left((2u)^2 - \left(\frac{11}{10}u\right)^2\right) + \frac{1}{2}\times 3m\left(u^2 - \left(\frac{8}{5}u\right)^2\right)$	M1 A(2,1,0)
	$= \frac{9}{20}mu^2$	A1 **(5)**
	(c) 	
	LM $\frac{24}{5}mu = 3ms + mt$	M1 A1
	NEL $t - s = \frac{8}{5}eu$	B1
	Solving to $s = \frac{2}{5}u(3-e)$	M1 A1
	For a further collision $\frac{11}{10}u > \frac{2}{5}u(3-e)$	M1
	$e > \frac{1}{4}$ ignore $e \le 1$	A1 **(7)**
		[17]

Q12.

Q	Scheme	Marks	Notes
a			
	CLM: $6mu = 2mv + 3mw$	M1	Requires all 3 terms. Must be dimensionally correct. Condone sign error(s)
	$(6u = 2v + 3w)$	A1	This equation defines their directions
	Impact: $w - v = \frac{3}{4} \times 3u \left(= \frac{9}{4}u \right)$	M1	Must be used with e on the correct side
		A1	Penalise inconsistent directions here
	$6u = 2w - \frac{9}{2}u + 3w$	DM1	Solve simultaneous equations for v or w Dependent on the 2 previous M marks
	$w = \frac{21}{10}u = v_B$	A1	One correct
	$v = w - \frac{9}{4}u = \left(\frac{21}{10} - \frac{9}{4}\right)u = -\frac{3}{20}u$, $v_A = -\frac{3}{20}u$	A1	Both correct
		(7)	
b	Speed of B after hitting wall $= \frac{21}{10}ue$	M1(B1)	$e \times$ their w
	Impulse $= \frac{27}{4}mu = 3m\left(\frac{21}{10}u + \frac{21}{10}ue\right)$	M1	for their w. Must be trying to use the correct equation with $3m$.
	$\frac{9}{4} = \frac{21}{10}(1+e)$, $e = \frac{1}{14}$	A1 (3)	
b alt	Impulse $= \frac{27}{4}mu = 3m\left(\frac{21}{10}u + V\right)$, $\left(V = \frac{3u}{20}\right)$	M1(B1)	Use impulse to find V. Must be trying to use the correct equation with $3m$.
	$\frac{21u}{10}e = \frac{3u}{20}$,	M1	$V = e \times$ their w .
	$e = \frac{1}{14}$	A1 (3)	
c	Speed of B after second impact = $\frac{1}{14} \times \frac{21}{10}u = \frac{3}{20}u$	B1ft	Compare two relevant speeds. (ft on their V or their e x their w)
	Same velocity (and A has a head start), so no collision.	B1 (2)	From correct work only
		[12]	

Q13.

Question Number	Scheme	Marks
(a)	KE lost : $\frac{1}{2}\times m\times 36-\frac{1}{2}\times m\times v^2=64$	M1A1
	Restitution: $v=$ 1/3 x 6 = 2	M1A1
	Substitute and solve for m: $\frac{1}{2}\times m\times 36-\frac{1}{2}\times m\times 4=64=16m$	DM1
	m = 4 answer given	A1
		(6)
(b)		
	Conservation of momentum: $6-8=4w-2v$ their "2"	M1A1ft
	Restitution: $v+w=⅔(2\ +\ 3)$ their "2"	M1A1ft
	$v=\frac{5}{3}-w$	
	Solve for w: $-2=4w-2(\frac{5}{3}-w)=6w-\frac{10}{3}$	DM1
	$\frac{4}{3}=6w$	A1
	$\left(w=4/18=2/9\,\text{m s}^{-1}\right)$	
	$w>\ 0\ \Rightarrow$ will collide with the wall again	A1
		(7)
		[13]

Q14.

Question Number	Scheme	Marks	Notes
a	CLM: $2mu = 2mv + 3mw$ Impact: $w - v = eu$ Subst $v = w - eu$: $2u = 2\ \ w - eu\ \ + 3w = 5w - 2eu$ $w = \frac{2}{5}\ 1 + e\ \ u$ ***Answer Given***	M1 A1 M1 A1 DM1 A1 (6)	All three terms required, but condone sign errors Condone sign error, but must be subtracting and e must be used correctly. Penalise inconsistent signs here. Solve for w. Requires the two preceding M marks
b	$w = \frac{7u}{10}$ CLM: $3mw = 3mx + 4my$ and Impact: $y - x = \frac{3w}{4}$ Subst: $3w = 3x + 4\left(x + \frac{3}{4}w\right)$ $x = 0$, $y = \frac{3}{4}w = \frac{21}{40}u$	B1 M1A1 DM1 A1 A1 (6)	Seen, or implied by correct speeds. Both needed Solve for x or y. Dependent on the preceding M mark $0.525u$,
c	$v = -\frac{u}{20}$ Speed of separation $= \frac{u}{20} + \frac{21u}{40} = \frac{23u}{40}$	B1 M1 A1 (3) [15]	Correct velocity of P Correct use of their values and substitute for e. Check directions carefully $0.575u$

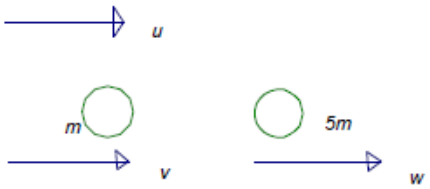
Q15.

Question Number	Scheme	Marks	Notes
(a)	$2mu = 2mv_P + mv_Q$	M1	CLM. Needs all 3 terms of corrwct form but condone sign slips
		A1	Correct equation
	$\frac{3}{4}mu^2 = \frac{1}{2}2mv_P^2 + \frac{1}{2}mv_Q^2$	M1	KE after impact. 3 terms of correct form
		A1	Correct equation
	$3v_Q^2 - 4uv_Q + u^2 = 0$ or $12v_P^2 - 16uv_P + 5u^2 = 0$	M1	Use CLM equation to form quadratic in v_P or v_Q
		A1	Correct equation
	$v_Q = \frac{u}{3}, v_P = \frac{5u}{6}$ or $v_Q = u, v_P = \frac{u}{2}$	DM1	Solve for a value of v_Q. Dependent on the previous M1.
		A1	A v_Q, v_P pair correct or two correct values for v_Q
	$v_Q = u$	DM1	Select solution from a choice of two. Dependent on all 4 M marks.
	..... since $v_Q > v_P$	A1	Correct justification
		(10)	
(b)	$e = \frac{u - \frac{u}{2}}{u}$ $\left(\frac{v_Q - v_P}{u}\right)$	M1	Impact law. Must be used correctly. Condone $\pm e$ Follow their speeds from (a).
		A1 ft	Correct for their speeds
	$= \frac{1}{2}$	A1	
		(3)	
		[13]	

Q16.

Question Number	Scheme	Marks	Notes
(a)	Speed after impact = $\frac{2}{3}u$ Impulse = change in momentum = $\pm\left(m.\text{their}\,\frac{2}{3}u - m.(-u)\right)\left(=\frac{5}{3}mu\right)$ $\lambda = \frac{5}{3}$	B1 M1 A1 (3)	Allow for velocity $= -\frac{2u}{3}$ Need to consider momentum before and after the collision and use change of direction. cso
(b)	Speed after second collision = $e^2u = \frac{4}{9}u$ Total time taken = $\frac{1}{u} + \frac{2}{eu} + \frac{1}{e^2u}\left(=\frac{1}{u}+\frac{3}{u}+\frac{9}{4u}\right)$	B1 M1 A2	Allow negative Use of time $= \frac{\text{distance}}{\text{speed}}$ to find the total time in terms of u. (At least one term dealt with correctly) -1 each error
Alt for M1A2	Ratio of times for the three sections is $\frac{1}{3}:1:\frac{3}{4}$	M1 A2	Or equivalent. -1 each error
	$\frac{4}{u}+\frac{9}{4u}=\frac{25}{4u}=3,\quad u=\frac{25}{12}$ o.e.	DM1 A1 (6)	Use total time = 3 and solve for u Accept 2.08 and 2.1 (or better)
		[9]	
	SC The candidate who only considers the first return to O can score the first M1 in (b) for $\frac{1}{u}+\frac{1}{eu}$ giving 1/6 marks		

Q17.

Question Number	Scheme	Marks
		
(a)	CLM: $mv + 5mw = mu$ NLI: $w - v = eu$ Solve v: $v = \frac{1}{6}(1-5e)u$, so speed $= \frac{1}{6}(5e-1)u$ **(NB – answer given on paper)** Solve w: $w = \frac{1}{6}(1+e)u$ * The M's are dependent on having equations (not necessarily correct) for CLM and NLI	B1 B1 M1* A1 M1* A1 (6)
(b)	After B hits C, velocity of B = "v" $= \frac{1}{6}(1-5.\frac{4}{5})u = -\frac{1}{2}u$ velocity $< 0 \Rightarrow$ change of direction $\Rightarrow$ B hits A	M1 A1 A1 CSO (3)
(c)	velocity of C after $= \frac{3}{10}u$ When B hits A, "u" $= \frac{1}{2}u$, so velocity of B after $= -\frac{1}{2}(-\frac{1}{2}u) = \frac{1}{4}u$ Travelling in the same direction but $\frac{1}{4} < \frac{3}{10}$ $\Rightarrow$ <u>no second collision</u>	B1 B1 M1 A1 CSO (4)

B1 Conservation of momentum – signs consistent with their diagram/between the two equations
B1 Impact equation
M1 Attempt to eliminate w
A1 correct expression for v. Q asks for speed so final answer must be verified positive with reference to e>1/5.
Answer given so watch out for fudges.
M1 Attempt to eliminate v
A1 correct expression for w

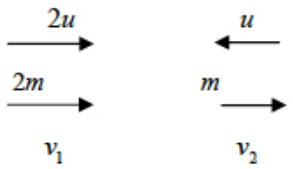
M1 Substitute for e in speed or velocity of P to obtain v in terms of u. Alternatively, can obtain v in terms of w
A1 (+/-) u/2 $(v = -\frac{5w}{3})$
A1 CSO <u>Justify direction</u> (and correct conclusion)

B1 speed of C = value of w = $(\pm)\frac{3u}{10}$ (Must be referred to in (c) to score the B1.)

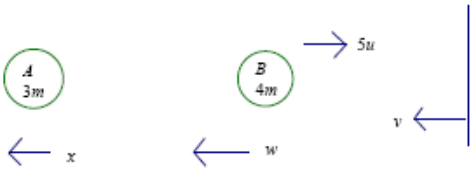
B1 speed of B after second collision $(\pm)\frac{1}{4}u$ or $(\pm)\frac{5}{6}w$

M1 Comparing their speed of B after 2nd collision with their speed of C after first collision.
A1 CSO. Correct conclusion .

Q18.

	 CLM: $4mu - mu = 2mv_1 + mv_2$ i.e. $3u = 2v_1 + v_2$ NIL: $3eu = -v_1 + v_2$ $v_1 = u(1-e)$ $v_2 = u(1+2e)$	M1 A1 M1 A1 DM1 A1 A1 **[7]**

Q19.

Q.	Scheme	Marks	Notes
a			
	Impact with wall: $v=\frac{3}{5}\times 5u=3u$	B1	or $-3u$
	Impulse $\pm 4m\left(3u-(-5u)\right)$	M1	M0 if clearly using $mv+mu$, otherwise bod
	Magnitude $32mu$ (Ns)	A1	
		(3)	
b	CLM: $3mx+4mw=4m\times 3u$	M1	Need all 4 terms. Condone sign errors. Use of $5u$ is M0
		A1ft	follow their $3u$
	Impact: $x-w=e\times 3u$	M1	Used the right way round. Use of $5u$ is M0
		A1ft	follow their $3u$ signs consistent with CLM equation
	$3m\left(w+3eu\right)+4mw=7mw+9emu=12mu$		
	$7w=u\left(12-9e\right)$	DM1	Solve for w or kw. Dependent on two preceding M marks
	Use of $e\le 1$ in their w: $7w\ge 3u$	M1	Condone use of $<$
	Hence $w>0$ and A and B are moving in the same direction	A1 (7)	Complete argument leading to ***given answer***
c	KE of B before collision $=\frac{1}{2}\times 4m\times\left(3u\right)^2\left(=18mu^2\right)$	B1	follow their $3u$. seen or implied
	$\Rightarrow\frac{1}{2}\times 4m\left(\frac{u}{7}\left(12-9e\right)\right)^2=\frac{1}{4}\left(\frac{1}{2}\times 4m\times 9u^2\right)$	M1	Follow their w. $\frac{1}{4}$ on the right side.
	$4\left(12-9e\right)^2=49\times 9,\ \left(4-3e\right)^2=\frac{49}{4}$	A1	Correct equation in m, u and e
	$e=\frac{1}{6}$	A1 (4)	
c alt	KE of B before collision $=\frac{1}{2}\times 4m\times\left(3u\right)^2\left(=18mu^2\right)$	B1	follow their $3u$
	$\Rightarrow\frac{1}{2}4mw^2=\frac{1}{4}\times\frac{1}{2}\times 4m\left(3u\right)^2\qquad\left(w=\frac{1}{2}\times 3u\right)$	M1	$\frac{1}{4}$ on the right side.
	$\frac{3}{7}\left(4-3e\right)=\frac{1}{2}\times 3$	A1	Correct equation in m, u and e from correct work only
	$e=\frac{1}{6}$	A1 (4)	0.17 or better from correct work only
		[14]	