In all the questions below, n is a positive integer.

17. If $2n$ is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

Let numbers be $2n$ and $2n+2$

$$(2n)^2 + (2n+2)^2$$

$$= 4n^2 + 4n^2 + 8n + 4$$

$$= 8n^2 + 8n + 4$$

$$= 4(2n^2 + 2n + 1)$$

4 is a factor so answer is a multiple of 4

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19. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

Let consecutive numbers be n and $n+1$

$$n^2 + (n+1)^2$$

$$= n^2 + n^2 + 2n + 1$$

$$= 2n^2 + 2n + 1$$

$$= 2n(n+1) + 1$$

Either n or $n+1$ must be even and so have a factor of 2.

$\therefore$ $2n(n+1)$ has a factor of 4

$\therefore$ $2n(n+1)+1$ = a multiple of 4 + 1

2. Prove that $(3n + 1)^2 – (3n – 1)^2$ is a multiple of 4, for all positive integer values of n.

$$(3n+1)^2 - (3n-1)^2$$

$$= (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$$

$$= \cancel{9n^2} + 6n \cancel{+1} - \cancel{9n^2} + 6n \cancel{-1}$$

$$= 12n$$

$$= 4(3n)$$

3. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

Let consecutive numbers be n and $n+1$

$n + n + 1 = 2n + 1$

which is odd

***5.** Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

Let integers be n and $n+1$

$$(n+1)^2 - n^2$$

$$= \cancel{n^2} + 2n + 1 - \cancel{n^2}$$

$$= 2n + 1$$

$$= n + (n+1) \quad \checkmark$$

Squaring Something and a half

Ex1 $\left(4\tfrac{1}{2}\right)^2 = 4 \times 5 + \tfrac{1}{4} = 20\tfrac{1}{4}$

Ex2 $\left(7\tfrac{1}{2}\right)^2 = 7 \times 8 + \tfrac{1}{4} = 56\tfrac{1}{4}$

$$\left(n + \tfrac{1}{2}\right)^2$$

$$= \left(n^2 + n + \tfrac{1}{4}\right)$$

$$= n(n+1) + \tfrac{1}{4}$$

Ex3 $\left(1\tfrac{1}{2}\right)^2 = 1 \times 2 + \tfrac{1}{4} = 2\tfrac{1}{4}$

Ex4 $25^2 = 625$

$(2 \cdot 5)^2 = 6 \cdot 25$

Ex 5 $\quad 45^2 = 2025$

Ex 6 $\quad 75^2 = 5625$

$\quad 95^2 = 9025$

Prime Numbers

A prime number is a positive integer which is only divisible by 1 and itself.

(1 is not regarded as a prime number)

Low Primes

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

31, 37, 41, 43, 47, 53, 59, 61, 67

71, 73, 79, 83, 89, 91, 97

All integers >1 can be written as the product of their prime factors

$$
\begin{array}{r|l}
2 & 72 \\ \hline
2 & 36 \\ \hline
2 & 18 \\ \hline
3 & 9 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}
$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$= 2^3 \times 3^2$$

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$60 = 2 \times 2 \times 3 \times 5$$

Find Highest Common Factor HCF of 60 and 72

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$\text{HCF} = 2 \times 2 \times 3 = 12$$

Find Lowest Common Multiple LCM of 72 and 60

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

Find HCF and LCM of 63 and 84

$$\begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$63 = 3 \times 3 \times 7$$

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$84 = 2 \times 2 \times 3 \times 7$$

HCF $= 3 \times 7 = 21$

LCM $= 2 \times 2 \times 3 \times 7 \times 3 = 249$
