Kinematics - Variable Acceleration

1. A particle P moves on the x-axis. The acceleration of P at time t seconds, $t \geq 0$, is $(3t + 5)$ m s^{-2} in the positive x-direction. When $t = 0$, the velocity of P is 2 ms^{-1} in the positive x-direction. When $t = T$, the velocity of P is 6 m s^{-1} in the positive x-direction. Find the value of T.

(Total 6 marks)

$\begin{cases} v = 2 \\ t = 0 \end{cases}$

$a = 3t + 5$

$v = \int a \, dt = \int (3t + 5) \, dt$

$v = \frac{3t^2}{2} + 5t + c$

$v = 2, t = 0$

$2 = 0 + 0 + c$

$2 = c$

$v = \frac{3t^2}{2} + 5t + 2$

$6 = \frac{3T^2}{2} + 5T + 2$

$12 = 3T^2 + 10T + 4$

$0 = 3T^2 + 10T - 8$

$0 = (3T - 2)(T + 4)$

$\Rightarrow \underline{T = \frac{2}{3}}$ or $\cancel{T = -4}$

2. A particle P moves along the x-axis. At time t seconds the velocity of P is v m s^{-1} in the positive x-direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity.

(Total 8 marks)

Displacement $x = \int v \, dt$

$x = \int (3t^2 - 4t + 3) \, dt$

$x = t^3 - 2t^2 + 3t + c$

$t=0,\ x=0 \Rightarrow c=0$

$$x = t^3 - 2t^2 + 3t$$

Min velocity when $\frac{dv}{dt} = a = 0$

$$\frac{dv}{dt} = 6t - 4$$

$$\frac{dv}{dt} = 0 \Rightarrow 6t - 4 = 0$$
$$6t = 4$$
$$t = \frac{2}{3} \text{ s}$$

Distance from O at time $t = \frac{2}{3}$

$$x = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)$$

$$x = \frac{8}{27} - \frac{8}{9} + 2$$

$$x = 1\tfrac{11}{27} \text{ m} \quad \text{or} \quad 1.41 \text{ m}$$

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3. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2.$$

(a) Find the maximum value of v.

(4)

(b) Find the time taken for P to return to O.

(5)
(Total 9 marks)

a) $\frac{dv}{dt} = 8 - 2t$

Max v when $\frac{dv}{dt} = 0 \Rightarrow 8 - 2t = 0$
$8 = 2t$
$t = 4$

$$\text{Max } v = 8(4) - 4^2 = 32 - 16 = 16 \text{ ms}^{-1}$$

b)

$$x = \int v\,dt = \int (8t - t^2)\,dt$$

$$= 4t^2 - \frac{t^3}{3} + c$$

$t = 0,\ x = 0 \Rightarrow c = 0$

$$x = 4t^2 - \frac{t^3}{3}$$

Returns when $x = 0$

$$0 = 4t^2 - \frac{t^3}{3}$$

$$0 = t^2\left(4 - \frac{t}{3}\right)$$

$$4 - \frac{t}{3} = 0$$

$$4 = \frac{t}{3}$$

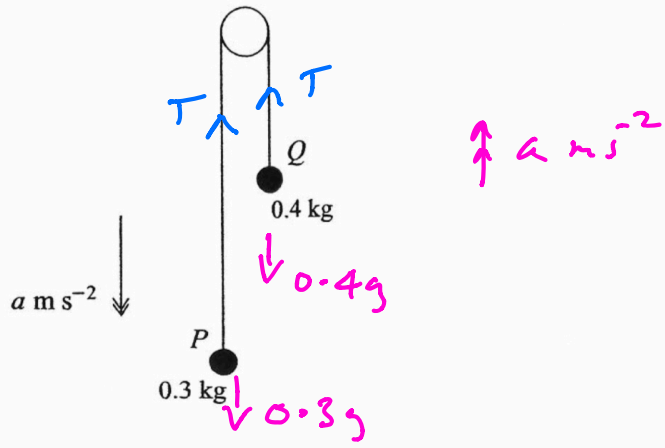
$$12 = t$$

$$t = 12 \text{ s}$$

Connected Particles

1

Jan '06



Particles P and Q, of masses 0.3 kg and 0.4 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is in motion with the string taut and with each of the particles moving vertically. The downward acceleration of P is a m s^{-2} (see diagram).

(i) Show that $a = -1.4$. [4]

Initially P and Q are at the same horizontal level. P's initial velocity is vertically downwards and has magnitude 2.8 m s^{-1}.

(ii) Assuming that P does not reach the floor and that Q does not reach the pulley, find the time taken for P to return to its initial position. [3]

i)

N2L for P $\quad 0.3g - T = 0.3a$

N2L for Q $\quad T - 0.4g = 0.4a$

Adding $\quad 0.3g - 0.4g = 0.7a$

$$\frac{-0.1g}{0.7} = a$$

$$a = -\frac{0.1 \times 9.8}{0.7}$$

$$a = -1.4 \text{ ms}^{-2}$$

ii)

$$s = ut + \tfrac{1}{2}at^2$$

+ve downwards

$$0 = 2.8t - \tfrac{1}{2} \times 1.4t^2$$

$$0.7t^2 - 2.8t = 0$$

$t(0.7t - 2.8) = 0$

$t = 0$ or $t = \frac{2.8}{0.7} = 4s$

Back at original position when $t = 4s$

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3 Fig. 3 shows a system in equilibrium. The rod is firmly attached to the floor and also to an object, P. The light string is attached to P and passes over a smooth pulley with an object Q hanging freely from its other end.

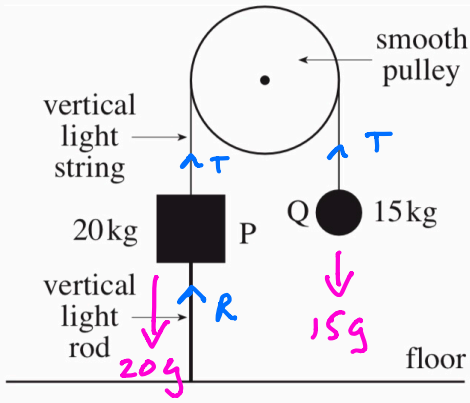

Fig. 3

(i) Why is the tension the same throughout the string? [1]

(ii) Calculate the force in the rod, stating whether it is a tension or a thrust. [3]

i) Smooth pulley

N2L for Q $T = 15g$

N2L for P $20g = R + T$

$20g = R + 15g$

$5g = R$

$R = 5g$ thrust