Sample Spaces

Monopoly Dice — Red and Blue

Blue

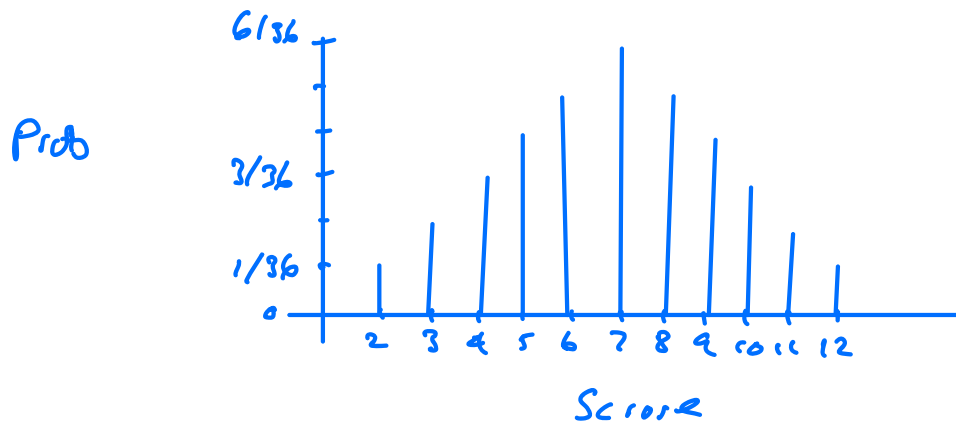| +	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Red

36 possible physical outcomes

A red 1 and a blue 2 is not the same outcome as a red 2 and a blue 1.

Probability Table

Score	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Find when you roll the two monopoly dice

1) $Prob(\text{Even score}) = \frac{18}{36} = \frac{1}{2}$

2) Prob (Score is prime) = $\frac{15}{36}$ = $\frac{5}{12}$

3) Prob(Score > 7) = $\frac{15}{36}$

4) Prob (Score a multiple of 3) = $\frac{12}{36}$ = $\frac{1}{3}$

5) Prob (Score same on each die) = $\frac{6}{36}$ = $\frac{1}{6}$

Multiplying the Scores on the Dice

X	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Which score has highest probability?

$$P(12) = \frac{4}{36} = \frac{1}{9}$$

2 × 6
6 × 2
3 × 4
4 × 3

Substract smaller score from larger score

−	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Which score has highest probability and what is it?

$$P(1) = \frac{10}{36} = \frac{5}{18}$$

Independent Events and the 'AND' Rule

Spinning a coin multiple times

Each spin is independent of the others and the prob of obtaining a Head remains at $\frac{1}{2}$

What is the probability of 3 successive heads

HHH
HHT
HTH
HTT
THH
THT

$$P(HHH) = \frac{1}{8}$$

T T H
T T T

$$P(HHH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

More generally if two events A and B are independent then

$$P(A \cap B) = P(A) \times P(B)$$

In other words the probability of two independent events happening is obtained by multiplying their individual probabilities together

Eg   Roll a die and spin a coin
     what is the prob of obtaining a 5 and a Head

$$P(5) = \frac{1}{6} \qquad P(H) = \frac{1}{2}$$

$$P(5 \cap H) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

H 1   T 1
H 2   T 2
H 3   T 3
H 4   T 4
(H 5)   T 5
H 6   T 6

$$P(5 \cap H) = \frac{1}{12}$$