Direct and Inverse Proportion

Direct Proportion

Let y be directly proportional to the square of x. When $x = 4$, $y = 48$.

a) Find a relationship between y and x

b) Find y when $x = 5$

c) Find x when $y = 300$

a)

$$y = kx^2$$

Sub $x=4$
$y=48$

$$48 = k \times 4^2$$

$$48 = 16k$$

$$\frac{48}{16} = k$$

$$3 = k$$

$$\therefore \underline{y = 3x^2}$$

b) when $x = 5$

$$y = 3 \times 5^2$$

$$y = 75$$

c) when $y = 300$

$$300 = 3x^2$$

$$100 = x^2$$

$$\sqrt{100} = x$$

$$x = \pm 10$$

Inverse Proportion

h is inversely proportional to the square root of q. When $q = 16$, $h = 3$

a) Find the relationship between h and q

b) Find h when $q = 4$

c) Find q when $h = 1$

a) $$h = \frac{k}{\sqrt{q}}$$

sub $q = 16$
$h = 3$

$$3 = \frac{k}{\sqrt{16}}$$

$$3 = \frac{k}{4}$$

$$3 \times 4 = k$$

$$12 = k$$

$$h = \frac{12}{\sqrt{q}}$$

b) When $q = 4$

$$h = \frac{12}{\sqrt{4}}$$

$$h = \frac{12}{2}$$

$$\underline{h = 6}$$

c) When $h = 1$

$$1 = \frac{12}{\sqrt{q}}$$

$$\sqrt{q} = 12$$

$$\underline{q = 12^2 = 144}$$

EXERCISE 22B

In each case, first find k, the constant of proportionality, and then the formula connecting the variables.

1 T is directly proportional to x^2. If $T = 36$ when $x = 3$, find the following.

a T when $x = 5$ **b** x when $T = 400$

2 W is directly proportional to M^2. If $W = 12$ when $M = 2$, find the following.

a W when $M = 3$ **b** M when $W = 75$

3 E varies directly with $\sqrt{C}$. If $E = 40$ when $C = 25$, find the following.

a E when $C = 49$ **b** C when $E = 10.4$

1)

$T = kx^2$

$T = 36$
$x = 3$

$36 = k \times 3^2$

$36 = 9k$

$\frac{36}{9} = k$

$4 = k$

$T = 4x^2$

a) when $x = 5$

$T = 4 \times 5^2$

$\underline{T = 100}$

b) when $T = 400$

$400 = 4x^2$

$\frac{400}{4} = x^2$

$100 = x^2$

$\sqrt{100} = x$

$x = \pm 10$

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2 W is directly proportional to M^2. If $W = 12$ when $M = 2$, find the following.

a W when $M = 3$ **b** M when $W = 75$

$W = kM^2$

$w = 12$
$M = 2$

$12 = k \times 2^2$

$12 = 4k$

$\frac{12}{4} = k$

$3 = k$

$W = 3M^2$

a) when $M = 3$

$W = 3 \times 3^2$

$W = 27$

b) when $W = 75$

$75 = 3M^2$

$\frac{75}{3} = M^2$

$25 = M^2$

$\sqrt{25} = M$

$M = \pm 5$

3 E varies directly with $\sqrt{C}$. If $E = 40$ when $C = 25$, find the following.

a E when $C = 49$

b C when $E = 10.4$

$E = k\sqrt{C}$

$E = 40$
$C = 25$

Sub
$E = 40$
$C = 25$

$40 = k \times \sqrt{25}$

$40 = 5k$

$\frac{40}{5} = k$

$8 = k$

$E = 8\sqrt{C}$

a) when $C = 49$

$E = 8\sqrt{49} = 8 \times 7$

$E = 56$

b) when $E = 10.4$

$10 \cdot 4 = 8\sqrt{c}$

$\frac{10 \cdot 4}{8} = \sqrt{c}$

$1 \cdot 3 = \sqrt{c}$

$1 \cdot 3^2 = c$

$c = 1 \cdot 69$

3 Q varies inversely with $(5 - t)$. If $Q = 8$ when $t = 3$, find the following.

a Q when $t = 10$

b t when $Q = 16$

4 M varies inversely with t^2. If $M = 9$ when $t = 2$, find the following.

a M when $t = 3$

b t when $M = 1.44$

5 W is inversely proportional to $\sqrt{T}$. If $W = 6$ when $T = 16$, find the following.

a W when $T = 25$

b T when $W = 2.4$

3) $Q = \frac{k}{5-t}$

Sub $Q = 8$
$t = 3$

$8 = \frac{k}{5-3}$

$8 = \frac{k}{2}$

$8 \times 2 = k$

$16 = k$

$Q = \frac{16}{5-t}$

a) When $t = 10$

$Q = \frac{16}{5-10}$

$Q = \frac{16}{-5}$

$Q = -3 \cdot 2$

b) when $Q = 16$

$16 = \frac{16}{5-t}$

$$16(5-t) = 16$$
$$80 - 16t = 16$$
$$80 - 16 = 16t$$
$$64 = 16t$$
$$\frac{64}{16} = t$$

$$\underline{t = 4}$$

4 M varies inversely with t^2. If $M = 9$ when $t = 2$, find the following.

a M when $t = 3$

b t when $M = 1.44$

$$M = \frac{k}{t^2}$$

Sub $M = 9$
$t = 2$

$$9 = \frac{k}{2^2}$$
$$9 = \frac{k}{4}$$
$$9 \times 4 = k$$
$$36 = k$$

$$\underline{M = \frac{36}{t^2}}$$

a) when $t = 3$ $\quad M = \frac{36}{3^2} = \frac{36}{9} = 4$

$$\underline{M = 4}$$

b) when $t = 1.44$

$$1.44 = \frac{36}{t^2}$$
$$1.44t^2 = 36$$
$$t^2 = \frac{36}{1.44}$$
$$t^2 = 25$$

$t = \sqrt{25}$

$t = \pm 5$

5 W is inversely proportional to $\sqrt{T}$. If $W = 6$ when $T = 16$, find the following.

a W when $T = 25$

b T when $W = 2.4$

$W = \frac{k}{\sqrt{T}}$

$6 = \frac{k}{\sqrt{16}}$

$6 = \frac{k}{4}$

$6 \times 4 = k$

$24 = k$

Sub $W = 6$
$T = 16$

$W = \frac{24}{\sqrt{T}}$

a) when $T = 25$

$W = \frac{24}{\sqrt{25}} = \frac{24}{5}$

$W = 4.8$

b) when $W = 2.4$

$2.4 = \frac{24}{\sqrt{T}}$

$2.4\sqrt{T} = 24$

$\sqrt{T} = \frac{24}{2.4} = 10$

$T = 10^2$

$T = 100$