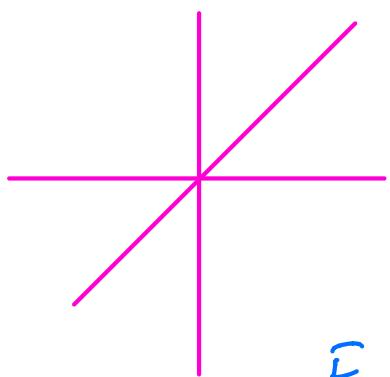


Invariant Lines

Reflection in Line $y=x$



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

Find invariant lines of transformation $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} x \\ mx+c \end{pmatrix}$$

$$0x + mx + c = x \quad (1)$$

$$x + 0 = -x + c \quad (2)$$

Solve for x in (2)

$$x = m(mx + c) + c$$

$$x = m^2x + mc + c$$

$$(1-m^2)x = c(m+1)$$

$$(1+m)(1-m)x = c(m+1)$$

has to be true for all x

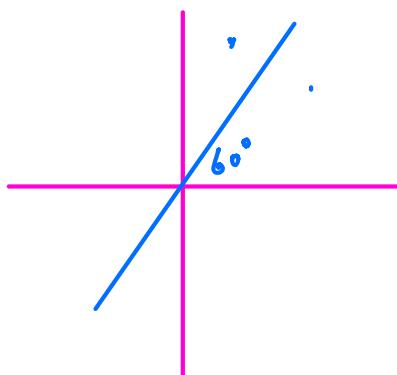
True when $c=0, m=\pm 1$

$$y=x, \quad y=-x$$

However, also true when $m = -1$
and c is unrestrained

$$y = -x + c$$

Ex 2 Reflection in line $y = \sqrt{3}x$



Equivalent to

1. Rotate 60° clockwise about O
2. Reflect in x -axis
3. Rotate 60° anti-clockwise about O

$$\begin{pmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos 60 & \sin 60 \\ -\sin 60 & \cos 60 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

Find invariant lines for transformation

$$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} x \\ mx+c \end{pmatrix}$$

$$-\frac{1}{2}x + \frac{\sqrt{3}}{2}(mx+c) = X$$

$$\frac{\sqrt{3}}{2}x + \frac{1}{2}(mx+c) = mX+c$$

$$-x + \sqrt{3}(mx+c) = 2X \quad (1)$$

$$\sqrt{3}x + mx+c = m2X + 2c \quad (2)$$

Sub for $2X$

$$\sqrt{3}x + mx+c = m(-x + \sqrt{3}(mx+c)) + 2c$$

$$x(\sqrt{3}+m)+c = m((\sqrt{3}m-1)x + \sqrt{3}c) + 2c$$

$$x(\sqrt{3}+m)+c = m(\sqrt{3}m-1)x + \sqrt{3}mc + 2c$$

$$x(\sqrt{3}+m) - m(\sqrt{3}m-1)x = \sqrt{3}mc + c$$

$$x \left[\sqrt{3} + m - \sqrt{3}m^2 + m \right] = c(1 + \sqrt{3}m)$$

$$x \left[\left(n - \sqrt{3} \right) \left(m + \frac{\sqrt{3}}{3} \right) \right] = c(1 + \sqrt{3}m)$$

$$c=0 \Rightarrow m = -\frac{\sqrt{3}}{3} \text{ or } \sqrt{3}$$

$$\underline{y = \sqrt{3}x} \quad \text{or} \quad y = -\frac{\sqrt{3}}{3}x$$

when $n = -\frac{\sqrt{3}}{3}$ c is unrestrained

so $y = -\frac{\sqrt{3}}{3}x + c$ is invariant

Find invariant lines of transformation

$$\begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix} \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} x' \\ mx'+c \end{pmatrix}$$

$$3x - 4(mx+c) = x'$$

$$-4x - 3(mx+c) = mx' + c$$

$$-4x - 3mx - 3c = n(3x - 4(mx+c)) + c$$

$$-4x - 3mx - 3c = 3nx - 4n^2x - 4nc + c$$

$$(4n^2 - 6n - 4)x + 4nc - 4c = 0$$

$$2(2n^2 - 3n - 2)x + 4c(n-1) = 0$$

$$2(2n+1)(n-2)x + 4c(n-1) = 0$$

$$m = 2, \angle = 0 \quad \text{or} \quad m = -\frac{1}{2}, \angle = 0$$

$$y = 2x$$

$$y = -\frac{1}{2}x$$

Hwts Find invariant lines of
stretch s.t 3 parallel to x -axis
and stretch s.t 2 parallel to y -axis

may
not
work

!!

$$\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Also
explore