

## Central Limit Theorem

### Summary of key points

1 The **central limit theorem** states that given a random sample of size  $n$  from any distribution with mean  $\mu$  and variance  $\sigma^2$ , the sample mean  $\bar{X}$  is approximately distributed as  $N\left(\mu, \frac{\sigma^2}{n}\right)$ .

1)  $X \sim N(10, 2^2)$

### Exercise 5A

$$Y \sim N\left(10, \frac{2^2}{6}\right)$$

$$Y \sim N\left(10, \sqrt{\frac{2^2}{6}}\right)$$

$$\begin{aligned} P(Y > 12) &= 1 - P(Y < 12) \\ &= 1 - 0.9928 \\ &= 0.0072 \end{aligned}$$

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Not an approximation since underlying dist is normal

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2)  $X \sim N(40, 1.5^2)$

a)  $Y \sim N\left(40, \frac{1.5^2}{4}\right)$

$$\begin{aligned} P(Y > 40.5) &= 1 - P(Y < 40.5) \\ &= 1 - 0.7475 \\ &= 0.2525 \end{aligned}$$

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b)  $Y \sim N\left(40, \frac{1.5^2}{49}\right)$

$$\begin{aligned} P(Y > 40.5) &= 1 - P(Y < 40.5) \\ &= 1 - 0.9902 \\ &= 0.0098 \end{aligned}$$

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### Exercise 5B

a)  $X \sim \text{Poisson}(3)$        $\mu = 3$   
 $\sigma^2 = 3$

b)  $\bar{X} \sim N\left(3, \frac{3}{10}\right)$

$P(\bar{X} < 2.5) = 0.1807$

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a)  $X \sim \text{Poisson}(3)$

$\bar{X} \sim \frac{1}{10} \text{Poisson}(30)$

$P(X < 25) = 0.2084$

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2)  $X \sim \text{Geo}(0.25)$        $\mu = \frac{1}{p} = \frac{1}{0.25} = 4$

$\bar{X} \approx N\left(4, \frac{12}{12}\right)$        $\sigma^2 = \frac{1-p}{p^2} = \frac{1-0.25}{0.25^2} = 12$

$P(\bar{X} > 5) = 1 - P(\bar{X} < 5)$

$= 1 - 0.8413$

$= 0.1587$

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