

Exercise 7C Factor Theorem

8) $(x-1)$ a factor of $5x^3 - 9x^2 + 2x + a$

$$f(x) = 5x^3 - 9x^2 + 2x + a$$

$$f(1) = 5(1)^3 - 9(1)^2 + 2(1) + a = 0$$

$$5 - 9 + 2 + a = 0$$

$$\underline{a = 2}$$

10) $(x-1), (x+1)$ factors of $f(x) = px^3 + qx^2 - 3x - 7$

$$f(1) = p(1)^3 + q(1)^2 - 3(1) - 7 = 0$$

$$\underline{p + q - 10 = 0} \quad ①$$

$$f(-1) = p(-1)^3 + q(-1)^2 - 3(-1) - 7 = 0$$

$$\underline{-p + q - 4 = 0} \quad ②$$

$$\begin{aligned} ① + ② & \quad 2q - 14 = 0 \\ & \quad 2q = 14 \\ & \quad q = 7 \end{aligned}$$

$$\begin{aligned} \text{Sub for } q \text{ in } ① & \quad p + 7 - 10 = 0 \\ & \quad p = 3 \end{aligned} \quad \left\{ \begin{array}{l} p = 3 \\ q = 7 \end{array} \right.$$

14) $f(x) = 4x^3 + 4x^2 - 11x - 6$

$$\begin{aligned} f(-2) &= 4(-2)^3 + 4(-2)^2 - 11(-2) - 6 \\ &= -32 + 16 + 22 - 6 \\ &= 0 \end{aligned}$$

$\therefore (x+2)$ is a factor of $f(x)$ by factor theorem

$$f(3) = 4(3)^3 + 4(3)^2 - 11(3) - 6 \\ \approx 108 + 36 - 33 - 6 > 0$$

$$f(-3) = 4(-3)^3 + 4(-3)^2 - 11(-3) - 6 \\ = -108 + 36 + 33 - 6 < 0$$

Abandon factor theorem!

$$\begin{array}{r} 4x^2 - 4x - 3 \\ \hline x+2 \left| \begin{array}{r} 4x^3 + 4x^2 - 11x - 6 \\ 4x^3 + 8x^2 \\ \hline -4x^2 - 11x \\ -4x^2 - 8x \\ \hline -3x - 6 \\ -3x - 6 \\ \hline \end{array} \right. \end{array}$$

$$f(x) = (x+2)(4x^2 - 4x - 3)$$

$$\begin{array}{r} 4x^3 \\ = -12 \\ +2 - 6 \\ \hline \end{array} \quad \begin{array}{r} 4x^2 - 4x - 3 \\ 4x^2 + 2x - 6x - 3 \\ 2x(2x+1) - 3(2x+1) \\ (2x-3)(2x+1) \end{array}$$

$$f(x) = (x+2)(2x-3)(2x+1)$$

$$f(x) = 0 \Rightarrow x = -2$$

$$x = \frac{3}{2}$$

$$x = -\frac{1}{2}$$

$$9) (x+3) \text{ factor of } f(x) = 6x^3 - 5x^2 + 18$$

$$\begin{aligned}f(-3) &= 6(-3)^3 - 6(-3)^2 + 18 = 0 \\&= -162 - 96 + 18 = 0 \\&\quad -144 = 9b\end{aligned}$$

$$\underline{b = -16}$$

$$11) (x+1), (x-2) \text{ factors of } f(x) = cx^3 + dx^2 - 9x - 10$$

$$\begin{aligned}f(-1) &= c(-1)^3 + d(-1)^2 - 9(-1) - 10 = 0 \\&= -c + d - 1 = 0 \quad \textcircled{1}\end{aligned}$$

$$\begin{aligned}f(2) &= c(2)^3 + d(2)^2 - 9(2) - 10 = 0 \\&= 8c + 4d - 28 = 0 \quad \textcircled{2}\end{aligned}$$

$$\text{From } \textcircled{1} \quad d = c + 1$$

$$\begin{aligned}\text{Sub for } d \text{ in } \textcircled{2} \quad 8c + 4(c+1) - 28 &= 0 \\8c + 4c + 4 - 28 &= 0 \\12c &= 24\end{aligned}$$

$$\underline{c = 2}$$

$$d = 2+1 = 3$$

$$\begin{cases} c = 2 \\ d = 3 \end{cases}$$

$$13) f(x) = 3x^3 - 12x^2 + 6x - 24$$

$$\begin{aligned}f(4) &= 3(4)^3 - 12(4)^2 + 6(4) - 24 \\&= 192 - 192 + 24 - 24\end{aligned}$$

$$= 0$$

By factor theorem $(x-4)$ is a factor of $f(x)$

$$x-4 \overline{)3x^3 - 12x^2 + 6x - 24}$$

$$\underline{3x^3 - 12x^2}$$

$$+ 6x - 24$$

$$\underline{+ 6x - 24}$$

$$f(x) = (x-4)(3x^2+6)$$

$$3x^2+6 > 6 \text{ for all } x$$

$\therefore x=4$ is only root of $f(x)=0$

15) $f(x) = 9x^4 - 18x^3 - x^2 + 2x$

$$f(2) = 9(2)^4 - 18(2)^3 - 2^2 + 2(2)$$

$$= 144 - 144 - 4 + 4$$

$$= 0$$

$\therefore (x-2)$ is a factor of $f(x)$

$$x-2 \overline{)9x^4 - 18x^3 - x^2 + 2x}$$

$$\underline{9x^4 - 18x^3}$$

$$-x^2 + 2x$$

$$\underline{-x^2 + 2x}$$

$$f(x) = (x-2)(9x^3-x)$$

$$= x(x-2)(9x^2-1)$$

$$f(x) = x(x-2)(3x+1)(3x-1)$$

Roots of $f(x) = 0$

$$x=0, \quad x=2, \quad x=-\frac{1}{3}, \quad x=\frac{1}{3}$$