

Basic Differentiation Homework Solutions

1. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{dy}{dx}$.

$$y = 8x^3 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$$

(Total 6 marks)

$$\frac{dy}{dx} = 24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}$$

$$\text{Or } \frac{dy}{dx} = 24x^2 - \frac{2}{\sqrt{x}} + 3 - \frac{2}{x^2}$$

Either form is acceptable

2. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$.

(Total 3 marks)

$$\frac{dy}{dx} = 4x^3 + \frac{1}{3}x^{-\frac{2}{3}}$$

3. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(4)

(b) Find an equation of the tangent to C at the point where $x = 2$

(4)

(Total 8 marks)

$$y = \frac{x^2 + 3x - 8x - 24}{x}$$

$$y = \frac{x^2 - 5x - 24}{x}$$

$$y = x - 5 - 24x^{-1}$$

$$a) \quad \frac{dy}{dx} = 1 + 24x^{-2}$$

$$\text{Or } \frac{dy}{dx} = 1 + \frac{24}{x^2}$$

$$b) \quad \text{When } x = 2, \quad \frac{dy}{dx} = 1 + \frac{24}{2^2} = 7$$

$$\text{When } x = 2, \quad y = \frac{(2+3)(2-8)}{2} = -15$$

Point on curve is (2, -15)

Tangent $y - y_1 = m(x - x_1)$

$$y - -15 = 7(x - 2)$$

$$y + 15 = 7x - 14$$

$$y = 7x - 14 - 15$$

$$y = 7x - 29$$

4. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

(a) $\frac{dy}{dx}$

(3)

$$y = 2x^3 + 3x^{-2}$$

$$\frac{dy}{dx} = 6x^2 - 6x^{-3}$$

$$\text{or } \frac{dy}{dx} = 6x^2 - \frac{6}{x^3}$$
