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3. $$f(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

a) $f(2) = \ln(2+2) - 2 + 1 = 0.386 > 0$

$f(3) = \ln(3+2) - 3 + 1 = -0.391 < 0$

$f(x)$ is continuous $\therefore$ $f(x) = 0$ for some $x \in (2,3)$

b) $x_1 = \ln(2.5+2) + 1 = 2.50408$

$x_2 = \ln(2.50408 + 2) + 1 = 2.50498$

$x_3 = \ln(2.50498 + 2) + 1 = 2.50518$

c) $f(2.5045) = \ln(4.5045) - 2.5045 + 1 = 5.77 \times 10^{-4} > 0$

$f(2.5055) = \ln(4.5055) - 2.5055 + 1 = -2.01 \times 10^{-4} < 0$

$\therefore$ $x = 2.505$ is root to 3 d.p.

since $2.5045 < x < 2.5055$

H 2 6 3 1 5 R B 0 6 2 4

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7.

$$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

a) $f(1.4) = 3(1.4)^3 - 2(1.4) - 6 = -0.568 < 0$

$f(1.45) = 3(1.45)^3 - 2(1.45) - 6 = 0.216 > 0$

$f(x)$ is continous $\therefore$ $f(x) = 0$ for some $x \in (1.4, 1.45)$

b) $3x^3 - 2x - 6 = 0$

$\Rightarrow \quad 3x^2 - 2 - \frac{6}{x} = 0$

$\Rightarrow \quad 3x^2 = 2 + \frac{6}{x}$

$\Rightarrow \quad x^2 = \frac{2}{3} + \frac{2}{x}$

$\Rightarrow \quad x = \sqrt{\frac{2}{x} + \frac{2}{3}}$

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Question 7 continued

$x_0 = 1.43$

c) $x_1 = \sqrt{\frac{2}{1.43} + \frac{2}{3}} = 1.4371$

$x_2 = \sqrt{\frac{2}{1.4371} + \frac{2}{3}} = 1.4347$

$x_3 = \sqrt{\frac{2}{1.4347} + \frac{2}{3}} = 1.4355$

d) $f(1.4345) = 3(1.4345)^3 - 2(1.4345) - 6 = -0.013 < 0$

$f(1.4355) = 3(1.4355)^3 - 2(1.4355) - 6 = 0.003 > 0$

$\therefore f(x) = 0$ for some $x \in (1.4345, 1.4355)$

$\therefore x = 1.435$ to 3 d.p.

Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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7.

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.
(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.
(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.
(3)

a) $f'(x) = 3xe^x + 3e^x$ (product rule)

At t.p. $f'(x) = 0 \Rightarrow 3xe^x + 3e^x = 0$

$3e^x(x+1) = 0$

$\Rightarrow x = -1$

$f(x) = -3e^{-1} - 1$

$P(-1, -3e^{-1} - 1)$

b) $x_{n+1} = \frac{1}{3}e^{-x_n}$ $\quad x_0 = 0.25$

$x_1 = \frac{1}{3}e^{-0.25} = 0.2596$

$x_2 = \frac{1}{3}e^{-0.2596} = 0.2571$

$x_3 = \frac{1}{3}e^{-0.2571} = 0.2578$

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Question 7 continued

c) $f(0.25755) = 3(0.25755)e^{0.25755} - 1 = -3.8 \times 10^{-4} < 0$

$f(0.25765) = 3(0.25765)e^{0.25765} - 1 = 1.09 \times 10^{-4} > 0$

$f(x)$ is continuous $\therefore$ $f(x) = 0$ for some $x \in (0.25755, 0.25765)$

$\therefore \quad x = 0.2576$ to 4 d.p.

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1.

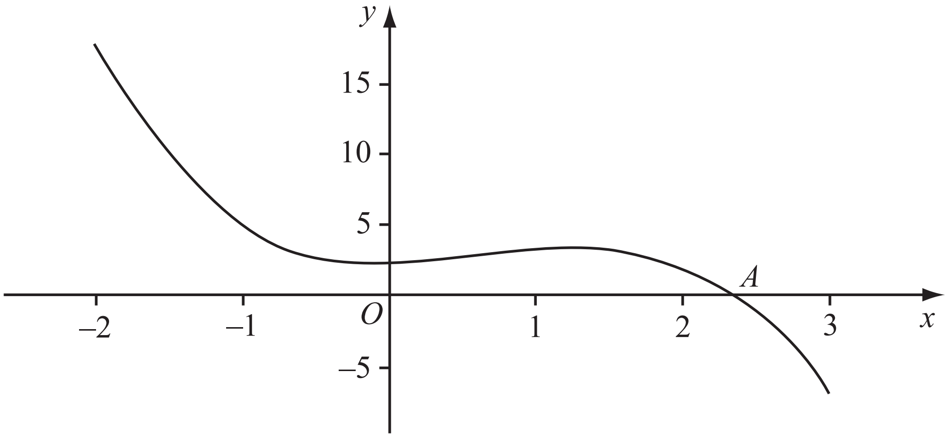

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.
(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.
(3)

a) $x_1 = \frac{2}{2.5^2} + 2 = 2.32$

$x_2 = \frac{2}{2.32^2} + 2 = 2.372$

$x_3 = \frac{2}{2.372^2} + 2 = 2.355$

$x_4 = \frac{2}{2.355^2} + 2 = 2.361$

b) $x = 2.3585, \Rightarrow y = -(2.3585)^3 + 2(2.3585)^2 + 2 = 5.8 \times 10^{-3} > 0$
$x = 2.3595, \Rightarrow y = -(2.3595)^3 + 2(2.3595)^2 + 2 = -1.4 \times 10^{-3} < 0$

Continuous function so $y = 0$ for some $x \in (2.3585, 2.3595)$
$\therefore x = 2.359$ to 3 d.p.

H 3 4 2 6 4 A 0 2 2 8

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2.

$$f(x) = x^3 + 2x^2 - 3x - 11$$

(a) Show that f(x) = 0 can be rearranged as

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

(2)

The equation f(x) = 0 has one positive root α.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$ is used to find an approximation to α.

(b) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)

a)

$$x^3 + 2x^2 - 3x - 11 = 0$$

$$x^3 + 2x^2 = 3x + 11$$

$$x^2(x+2) = 3x + 11$$

$$x^2 = \frac{3x+11}{x+2}$$

$$x = \sqrt{\frac{3x+11}{x+2}}$$

b) $x_1 = 0$

$$x_2 = \sqrt{\frac{0+11}{0+2}} = 2.345$$

$$x_3 = \sqrt{\frac{3(2.345)+11}{2.345+2}} = 2.037$$

$$x_4 = \sqrt{\frac{3(2.037)+11}{2.037+2}} = 2.059$$

N 3 5 3 8 1 A 0 4 2 8

c) $f(2.0565) = 2.0565^3 + 2(2.0565)^2 - 3(2.0565) - 11$

$= -0.014 < 0$

$f(2.0575) = 2.0575^3 + 2(2.0575)^2 - 3(2.0575) - 11$

$= 4.14 \times 10^{-3} > 0$

$f(x)$ is continuous so $f(x) = 0$ for some $x \in (2.0565, 2.0575)$

$\therefore$ root is $x = 2.057$ to 3 d.p.

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3. $\mathrm{f}(x) = 4\operatorname{cosec} x - 4x + 1$, where x is in radians.

(a) Show that there is a root α of $\mathrm{f}(x) = 0$ in the interval $[1.2, 1.3]$.
(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$
(2)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \qquad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.
(3)

(d) By considering the change of sign of $\mathrm{f}(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.
(2)

a) $f(x) = \dfrac{4}{\sin x} - 4x + 1$

$f(1.2) = \dfrac{4}{\sin(1.2)} - 4(1.2) + 1 = 0.492 > 0$

$f(1.3) = \dfrac{4}{\sin(1.3)} - 4(1.3) + 1 = -0.049 < 0$

$f(x)$ is continuous on $[1.2, 1.3]$

$\therefore f(x) = 0$ for some $x \in (1.2, 1.3)$

b) $4\operatorname{cosec} x - 4x + 1 = 0$

$\dfrac{4}{\sin x} + 1 = 4x$

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Question 3 continued

$$\frac{1}{\sin x} + \frac{1}{4} = x$$

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

c) $x_0 = 1.25$

$$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4} = 1.3038$$

$$x_2 = \frac{1}{\sin(1.3038)} + \frac{1}{4} = 1.2867$$

$$x_3 = \frac{1}{\sin(1.2867)} + \frac{1}{4} = 1.2918$$

d) $f(1.2905) = \frac{4}{\sin(1.2905)} - 4(1.2905) + 1 = 0.114 > 0$

$f(1.2915) = \frac{4}{\sin(1.2915)} - 4(1.2915) + 1 = -4.7 \times 10^{-3} < 0$

$f(x)$ is continuous in this region so $f(x) = 0$

for some $x \in (1.2905, 1.2915)$

$\therefore$ root is $x = 1.291$ to 3 d.p.