Leave blank

7. The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

If the lines l_1 and l_2 intersect, find

(a) the value of α,

(4)

(b) an equation for the plane containing the lines l_1 and l_2, giving your answer in the form $ax + by + cz + d = 0$, where a, b, c and d are constants.

(4)

For other values of α, the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(c) find the shortest distance between the lines l_1 and l_2.

(3)

a) $\underline{r}_1 = \begin{pmatrix} 1-\lambda \\ -1+3\lambda \\ 2+4\lambda \end{pmatrix}$ $\underline{r}_2 = \begin{pmatrix} \alpha \\ -4+3\mu \\ 2\mu \end{pmatrix}$

From (3) $2 + 4\lambda = 2\mu$

$1 + 2\lambda = \mu$

Sub in (2) $-1 + 3\lambda = -4 + 3(1 + 2\lambda)$

$-1 + 3\lambda = -4 + 3 + 6\lambda$

$0 = 3\lambda$

$0 = \lambda$

$1 = \mu$

$\alpha = 1 - \lambda$

$\alpha = 1 - 0$ $\underline{\alpha = 1}$

Point of intersection $(1, -1, 2)$

M 3 5 1 4 5 A 0 2 0 2 8

Leave blank

Question 7 continued

b) Let normal be $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$\perp$ to l_1 and l_2

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} = 0 \qquad -a + 3b + 4c = 0$ *

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} = 0 \qquad 3b + 2c = 0$

$c = -\frac{3b}{2}$

Sub in * $\qquad -a + 3b - 6b = 0$

$a = -3b$

normal $= \begin{pmatrix} -3b \\ b \\ -\frac{3b}{2} \end{pmatrix}$ or $\begin{pmatrix} -3 \\ 1 \\ -\frac{3}{2} \end{pmatrix}$ or $\begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix}$

Plane $\qquad -6x + 2y - 3z + d = 0$

$(1, -1, 2)$ on plane $\qquad -6(1) + 2(-1) - 3(2) + d = 0$

$-6 - 2 - 6 + d = 0$

$d = 14$

$-6x + 2y - 3z + 14 = 0$

The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} \cancel{\alpha}\ 2 \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

General Points $\begin{pmatrix} 1-\lambda \\ -1+3\lambda \\ 2+4\lambda \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -4+3\mu \\ 2\mu \end{pmatrix}$

Let closest points be A and B

$$\overrightarrow{AB} = \begin{pmatrix} 2-(1-\lambda) \\ -4+3\mu-(-1+3\lambda) \\ 2\mu-(2+4\lambda) \end{pmatrix} = \begin{pmatrix} 1+\lambda \\ -3+3\mu-3\lambda \\ -2+2\mu-4\lambda \end{pmatrix}$$

When $\overrightarrow{AB}$ $\perp$ to both lines

$$\begin{pmatrix} 1+\lambda \\ -3+3\mu-3\lambda \\ -2+2\mu-4\lambda \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} = 0$$

$$-1-\lambda-9+9\mu-9\lambda-8+8\mu-16\lambda=0$$

$$-18-26\lambda+17\mu=0 \qquad (1)$$

$$\begin{pmatrix} 1+\lambda \\ -3+3\mu-3\lambda \\ -2+2\mu-4\lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} = 0$$

$$-9+9\mu-9\lambda-4+4\mu-8\lambda=0$$

$$-13-17\lambda+13\mu=0 \qquad (2)$$

$$\lambda = -\frac{13}{49} \qquad \mu = \frac{32}{49}$$

$$\overrightarrow{OA} = \begin{pmatrix} \frac{62}{49} \\ -\frac{88}{49} \\ \frac{46}{49} \end{pmatrix} \qquad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -\frac{100}{49} \\ \frac{64}{49} \end{pmatrix}$$

$$|AB| = \sqrt{\left(2 - \frac{62}{49}\right)^2 + \left(-\frac{100}{49} - -\frac{88}{49}\right)^2 + \left(\frac{64}{49} - \frac{46}{49}\right)^2}$$

$$= \sqrt{\left(\frac{36}{49}\right)^2 \left(-\frac{12}{49}\right)^2 + \left(\frac{18}{49}\right)^2}$$

$$= \frac{42}{49}$$

Leave blank

7. The plane Π has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

(a) Find an equation of Π in the form $\mathbf{r}.\mathbf{n} = p$, where $\mathbf{n}$ is a vector perpendicular to Π and p is a constant.

(5)

The point P has coordinates $(6, 13, 5)$. The line l passes through P and is perpendicular to Π. The line l intersects Π at the point N.

(b) Show that the coordinates of N are $(3, 1, -1)$.

(4)

The point R lies on Π and has coordinates $(1, 0, 2)$.

(c) Find the perpendicular distance from N to the line PR. Give your answer to 3 significant figures.

(5)

Leave blank

Question 7 continued

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6. The plane P has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$$

(a) Find a vector perpendicular to the plane P.

(2)

The line l passes through the point A (1, 3, 3) and meets P at (3, 1, 2).

The acute angle between the plane P and the line l is α.

(b) Find α to the nearest degree.

(4)

(c) Find the perpendicular distance from A to the plane P.

(4)

Leave blank

Question 6 continued

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3. The position vectors of the points A, B and C relative to an origin O are $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $7\mathbf{i} - 3\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j}$ respectively.

Find

(a) $\overrightarrow{AC} \times \overrightarrow{BC}$,

This is a vector product and not on Core Pure syllabus

(4)

(b) the area of triangle ABC,

(2)

(c) an equation of the plane ABC in the form $\mathbf{r}.\mathbf{n} = p$

(2)

P 4 0 1 1 1 A 0 6 3 2

Leave blank

Question 3 continued