Divisibility Tests by Induction

Ex Prove by induction $n^3 - 7n + 9$ is divisible by 3 for all +ve integers

$n = 1$ $\quad 1^3 - 7(1) + 9 = 3$ divisible by 3 ✓

Assume true for $n = k$

$\therefore \quad k^3 - 7k + 9 = 3h$ for some integer h

Consider $n = k+1$

then $(k+1)^3 - 7(k+1) + 9$

$$= k^3 + 3k^2 + 3k + 1 - 7k - 7 + 9$$

$$= k^3 - 7k + 9 + 3k^2 + 3k - 7 + 1$$

$$= 3h + 3k^2 + 3k - 6$$

$$= 3(h + k^2 + k - 2)$$

which is divisible by 3 since 3 is a factor

$\therefore$ if formula true for $n = k$, also true for $n = k+1$. Since true for $n = 1$, by mathematical induction it is true for all positive integers n

Exercise 8B

6) Prove $2^{6n} + 3^{2n-2}$ is divisible by 5

$n=1$ $2^6 + 3^0 = 64+1 = 65$ divisible by 5 ✓

True for $n=1$

Assume true for $n=k$

then $2^{6k} + 3^{2k-2} = 5h$ for some integer h

Consider $n=k+1$

$$2^{6(k+1)} + 3^{2(k+1)-2}$$

$$= 2^{6k} \times 2^6 + 3^{2k+2-2}$$

$$= 64 \times 2^{6k} + 3^2 \times 3^{2k-2}$$

$$= 64 \times 2^{6k} + 9 \times 3^{2k-2}$$

$$= 55 \times 2^{6k} + 9\left(2^{6k} + 3^{2k-2}\right)$$

$$= 55 \times 2^{6k} + 9(5h)$$

$$= 5\left(11 \times 2^{6k} + 9h\right)$$

Note $\left(11 \times 2^{6k} + 9h\right)$ is an integer

Divisible by 5 since 5 is a factor

$\therefore$ if true for $n=k$, true for $n=k+1$ etc

4) Prove $8^n - 3^n$ divisible 5 for all positive integers n

$n=1$ $\quad 8^1 - 3^1 = 8 - 3 = 5$ ✓

divisible by 5

Assume true for $n=k$

then $8^k - 3^k = 5h$ for some integer h

Consider $n = k+1$

$$8^{k+1} - 3^{k+1} = 8 \times 8^k - 3 \times 3^k$$
$$= 5 \times 8^k + 3 \times 8^k - 3 \times 3^k$$
$$= 5 \times 8^k + 3(8^k - 3^k)$$
$$= 5 \times 8^k + 3(5h)$$
$$= 5(8^k + 3h)$$

divisible by 5

$\therefore$ if true for $n=k$ also true for $n=k+1$ etc

Hwk: Spend an hour on Exercise 8B